A Model for Product Mix Planning Using Multi-objective Optimization Approach with Sensitivity Analysis

Ma. Teodora E. Gutierrez* and Philip P. Ermita

***Abstract*—The paper proposes a model for allocation problems in business organizations considering multiple objectives. It concerns the distribution of resources among options in order to achieve the best value for the goals of the business. Thus, the paper develops a model for a product mix plan using the multi-objective optimization (MOO) approach. The objectives set by the paper are to minimize production costs and waste. The inclusion of the production wastes was motivated by United Nations Sustainable Development Goal number 13, which states, "Take immediate action to combat climate change and its impacts." The developed model was then tested in a manufacturing company and resulted in Pareto-optimal solutions. Such solutions determine the quantity of products to be produced in several production facilities at a particular period of time and will be set as the product mix plan of the case study company. A sensitivity analysis was also performed by varying the weights of the objective functions, which resulted in different decisions about the best product mix plan. It was found that solution 2 is more stable than solution 1. Moreover, the paper also analyzed the permissible ranges for the coefficients of the decision variables and the coefficients for the constraints, where the optimal solutions remain unchanged. The proposed mathematical model thus demonstrates its viability and usefulness in achieving the company's aims.**

***Index Terms*—Multi objective optimization (MOO), pareto optimal, sustainable development goals (SDGs), product mix planning, sensitivity analysis**

I. INTRODUCTION

The concept of sustainable development was formally introduced in UN report Our Common Future [1]. Likewise, in year 2000, the United Nations Development Programme (UNDP) provides a development paradigm about sustainable development. In 2015, the Paris Agreement by 196 member parties committed to transformed the trajectory of sustainability to limit global warming to below 20C, above pre –industrial levels [2]. There is a strong concern for implementing sustainable development in all business activities and human activities. Sustainable development is defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" [3]. Resource scarcity and environmental pollution have increasingly a major concern globally. Strict regulations and the demand for innovations have driven organizations to implement sustainable practices [4]. Manufacturing is a critical contributor to a country's economic growth, consequently, the integration of sustainable production systems should be implemented in the complete life cycle of the products [5]. Sustainable development is link to manufacturing and consumption of both goods and services to reduce production waste. Organizations are forced to integrate reduction of production waste as part of their operational strategies. Motivating factors such as regulation [6] and evolving environmental criteria as basis for being competitive [7] are the considered now as one of the manufacturing strategies. The future product demands are linked to the manufacturing and consumption of goods and services, which lead to sustainable development and reduction of production waste. This paper supports business goals of having a sustainable production system which considers the achievement of two objectives, the production costs and the production waste. Furthermore, this project helps contributes to the achievement of Sustainable Development Goals (SDG) number 13, which stated as "Take urgent action to combat climate change and its impacts".

The paper aims to develop a mathematical model for the optimum mix of products to produce that considers two objective functions which are to minimize the production cost and at the same time, to minimize the waste of the production system.

Manuscript received August 31, 2022; revised February 3, 2023; accepted March 12, 2023.

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II. LITERATURE REVIEW

The United Nations Environment Programme (UNEP), develops the 2030 Agenda for Sustainable Development, where one of the goals is to reduce the use of natural resources as production materials and reduce the emission of wastes and pollutions over the life cycle of the products [8]. As more and more customers are purchasing sustainable products, companies need to adopt to new sustainable practices. The increasing deterioration of our environment forced stakeholders to include sustainability in their business goals. An adoption of corporate standard for corporate environmental management was release in 2005 [9]. The United Nations goal is to limit the use of greenhouse gas emission among highly industrialized countries. Moreover, one of the key elements for achieving sustainable development is the transition towards Sustainable Consumption and Production [8]. Manufacturing plays a critical role in our economy, with this, the integration of sustainable production practices increasingly being considered by a great number of manufacturing firms.

In the Philippines several laws and decrees were enacted to address environmental concerns. One of which is the Clean Air Act in 1999, which provides for a comprehensive air pollution control policy and a national programmed to prevent, manage, control, and reverse air pollution through both regulatory and market-based instruments. Several

doi: 10.18178/ijimt.2023.14.4.952

experts stated that for better implementation of less pollution in production, investigation should be conducted [10], especially in improving resources efficiency for reduction of environmental impacts [11]. Hence, this study will fill the said gap.

III. Methods

The case study company has three major products that can produce in any of their three production facilities. The company wants to minimize the production costs and the production waste. The proposed solution technique is a systematic process that minimizes production cost and also at the same time minimizes production waste which will result to a sustainable production system. A multi objective optimization model is propose. This sustainable production system model helps business organizations improved their performance on production cost and production waste

The first objective is to minimize the cost within the production. The production costs are assumed to be fixed production costs and variable costs. The second objective is the environmental concern which is minimizing the waste within the production. Since there are two (2) different objectives with different units of magnitude, a multi-objective optimization technique was used. The weighted-sum approach was used. Below is the proposed mathematical model for sustainable production systems.

The procedure to formulate a mathematical model for multi objective optimization are, first step is to identify the indices and notations for proper reference, second step is to identify the input parameters or the data to be collected. The third step is to identify the decision variables, which is to find the optimum number of products to produce and the facilities assigned to produce the products. The fourth step is to identify the objectives of the case study company. The fifth step is to consider the constraints or limitations of the case study company. The last step is to solve using the multi objective optimization approach, which can be coded in Linear programming model software.

A. Indices and Notations

j represents production facility
i represents product
r represents production waste

B. Input Parameters

Aij = Fixed production cost for product i at production facility j

Bij = Variable cost for product i at production facility j

Rij= Production waste of production facility j per unit of product i

Gik = Production Capacity of production facility j to produce product i

Oi = Total demand of product i

C. Decision Variables

Let:

Yij =1, *if product i is produced at production facility j*

Yij =0, *if product i is not produced at facility j*

Xij =Quantity of product i to produce in production facility j

D. Objective Functions

The objective is to minimize the production costs and production wastes of the firm.

1) Economic objective

The economic objective is to minimize the cost within the production. The production costs are assumed to be fixed production cost and variable cost. That is, Minimize Total Cost (Z1) = Fixed Cost + Variable Cost

$$Min\ Z1=\sum_{i=1}^{I} Aij\ x\ Yij + \sum_{i=1}^{I}\sum_{j=1}^{J} Bij\ x(Xij\ \ x\ Yij) \quad (1)$$

2) Environmental objective

The environmental objective is to optimize by minimizing the waste within the production. That is,
Minimize (Z2) =Production waste

$$Min\ Z2= \sum_{i=1}^{I}\sum_{j=1}^{J} Rij\ x\ (Xij\ \ x\ Yij) \quad (2)$$

E. Multi-objective Optimization

Since there are two (2) different objectives with different units of magnitude, a multi- objective optimization technique was used with weighted-sum approach. The general form for this technique is,

$$Min\ Q = w_1*(Z1) + w_2*(Z2) \quad (3)$$

The equation 3 is for the multi objective functions to become a single objective function. The weights (w_i) are set for economic performance (Z1) and the environmental performance (Z2). The single function objective of a multi objective function is to minimize the deviations from the target value.

In order to remove the units of the two objective functions, thus, to become single objective function, we will use the equation below,

Minimization Q (deviations) = w1*(actual value-target value)/target value + w2*(actual value-target value)/ target value (4)

F. Constraints

After determining the objectives, the study considers the general constraints of the problem.

$$Xij <=Gik\ x\ Yij,\ \forall j \quad (5)$$

$$Xij>=Oi \quad (6)$$

$$Xij>=0\ and\ integer \quad (7)$$

$$Yij= 0, 1 \quad (8)$$

The first constraint, equation (5) is about the number of units of product i to be produce should not exceed the capacity of each production facilities j. The second constraint, equation (6), is about the number of units of product k to produce should be greater than or equal to the total product demand. The third constraint, equation (7) is about the decision variables to have an integer value or whole number digits. And the last constraint, equation (8), is the decision variables to have a binary value, which is to have only two numbers, either zero (0) or one (1).

IV. Results and Discussions

From the mathematical model formulation above, the data collected from the case study company were plug in the equations. These mathematical model was coded in linear programming software to compute for the decision variables.

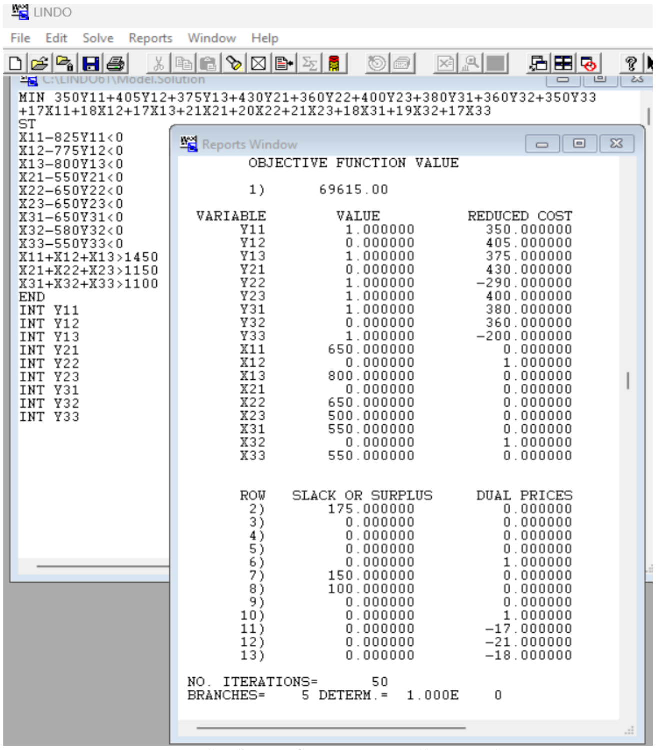


Fig. 1. Optimal solution for economic objective(Min z1).

Fig. 1 shows the optimal solutions for minimization of production costs (Min Z1). The goal of the algorithm is to solve one objective function at a time. Thus, the first line in the software worksheet is the minimization of the production cost, which consists of fixed costs and variable costs, followed by all the constraints as stated previously. Then an optimal solution was found, which was marked as Solution 1. This solution 1 was the result of a mathematical model in which the objective function, minimize product cost (Min Z1), was solved first. The results are that 650 units of product 1 should be produced in facility 1 (X11 = 650) and 800 units of product 1 should also be produced in facility 3 (X11 = 800). Also, 650 units of product 2 should be produced in facility 2 (X22 = 650). Product 2 should also be produced in facility 3 by 500 units (X23 = 500). For product 3, it should be produced in facility 1 (X31 = 550) and facility 3 (X33 = 550). This will give the lowest production cost of 69,615 Philippine Pesos (Php) (Min Z1 = 69,615).

On the other hand, Fig. 2 shows the optimal solutions for minimization of production wastes (Min z2) in grams. The results are as follows; product 1 should be produce in facility 1 by 850 units (X11=825) also in facility 3 (X13=625), product 2 should be produce in facility 2 and facility 3 respectively (X22 = 500, X23=650). For product 3 it should be produced in facility 1 and facility 3 by 550 units and 550 units, respectively (X31 = 550, X33=550). The outcome is the lowest production waste which value at 15,297 grams (Min Z2=15297).

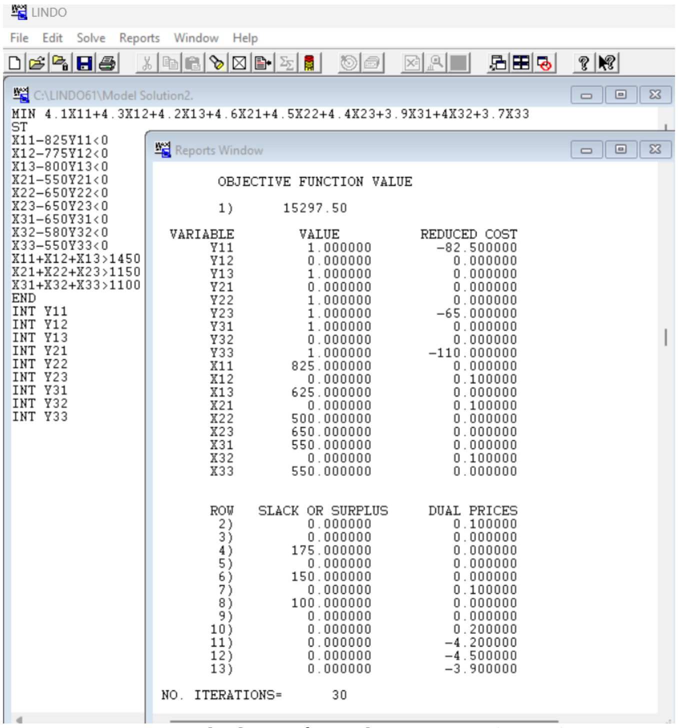


Fig. 2. Optimal solutions for production wastes (Min z2).

TABLE I: Summary of Optimal Solutions

Decision Variables	Solution 1	Solution 2
X11	650	825
X12	0	0
X13	800	625
X21	0	0
X22	650	500
X23	500	650
X31	550	550
X32	0	0
X33	550	550
Z1 (Cost)	69615	69765
Z2 (Waste)	15330	15297
Min Q	0.10786%	0.10774%

Table I shows the summary of the optimal solutions. To get the total production wastes for solution 1, equation (2) was used and the calculated value is 15,330 grams. On the other hand, the production costs for solution 2, equation (1) was used and the calculated value is 69,765 Philippine pesos. Solution 1 and Solution 2 are the options for the product mix plan of the company. To determine which options is best, multiple objective optimizations was applied with a formulated single function objective as stated in equation (3). This single function objective is to minimize the deviation from the target or ideal value which is notated as Minimize Q (Min Q). The study uses the earlier stated equations (3) and equation (4) to measure the Min Q of solution 1. The w denotes for the weights or percentage of importance of the objective function. The study uses 0.50 for objective 1 (Z1) and 0.50 for objective 2 (Z2), it means both objectives are the same percentage of importance. The resulted objective values for solution 1 and solution 2 are considered as target values since this is the optimum solutions for each objective. The computed sum of the weighted percent deviation for solution 1 is 0.10786 %. The computations are as follows:

Min Q = w [Z1] + w[Z2]

Min Q (deviations) =w_1*(actual value-target value)/(target value) + w_2*(actual value-target value)/ target value

Min Q (Sum of weighted % deviation) = 0.5 [69615-69615/69615] + 0.5[15330-15297/15297]

Min Q = .1079%

On the other hand, to compute for the Min Q of solution 2, the same equations are used, below are the computations,

Min Q (Sum of weighted % deviation) = 0.5 [69615-69765/69615] + 0.5[15297-15297/15297]

Min Q = .1077%

The computed sum of the weighted percent deviation for solution 2 is .10774%. Comparing the two-product mix plan (i.e., Solution 1 and Solution 2), solution 2 has the lowest sum of the percentage deviation, hence, it was chosen that solution 2 is the optimal solution for these multiple objectives problem. The product mix plan of solution 2 will result in the achievement of a more sustainable production plan.

The paper also tests the robustness of the solution 2 by changing the weights of the objective functions. This is called sensitivity analysis or what if scenario. It shows how changes in objective weights also changes the optimal solutions for the product mix plan. Fig. 3 shows varied weights of the objective functions. In the figure, weights in environmental objectives represented by orange bar. While the blue bar is represented by the weights of the economic objectives. The gray bar represents the values of the deviation from the target values in percentage. The desirable value for gray bar is at the minimum. For instance, if there is a greater weight of economic objective, the solution 1 is the optimum product mix plan. Whereas, if the environmental objective has greater weights than the economic objective, the solution 2 is the optimum product mix plan.

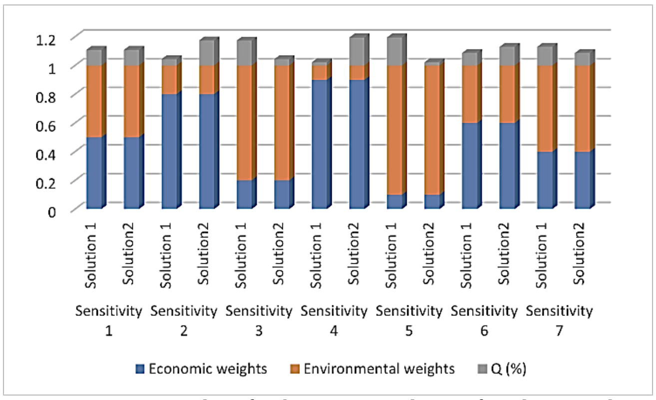
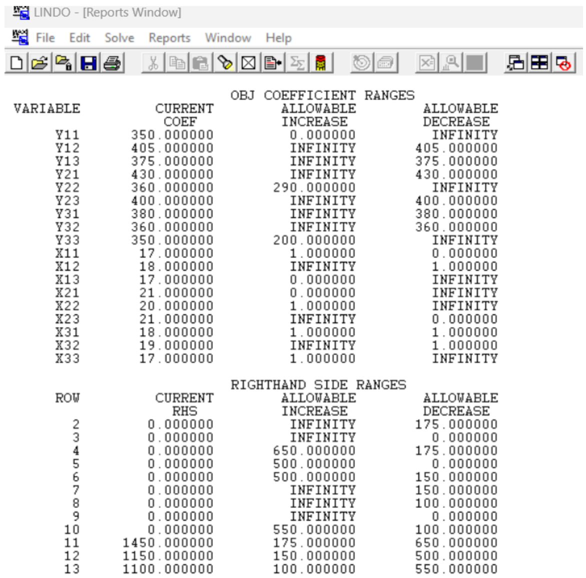


Fig. 3. Sensitivity analysis for the optimum solution of product mix plan.

Fig. 4 shows the permissible ranges for the current coefficients of the decision variables and for the current coefficients of the constraints where the optimal solution remains unchanged. For instance for the notation y11 which denotes for the fixed cost of facility 1 to produce product 1, the allowable increase is 0 and the allowable decrease is infinity. It means thay the solution remains unchanged for any decrease of the current fixed cost and at the same time no increase of the current fixed cost. Hence, the allowable range of the value for y11 is from 0 to 350 only. For the right hand side ranges, row 11 of the LINDO model, which is the demand constraints, the allowable changes of the current demand is an increase of 175 units and a decrease of 450 units, where the optimal solution is unchanged. Or, the allowable ranges for the demand of product 1 is from 800 units to 1625 units. Likewise, Fig. 5 shows the allowable ranges of the coefficients of decision variables and the coefficients of the contraints , where the optimal solutions for environmental objective remains unchanged.



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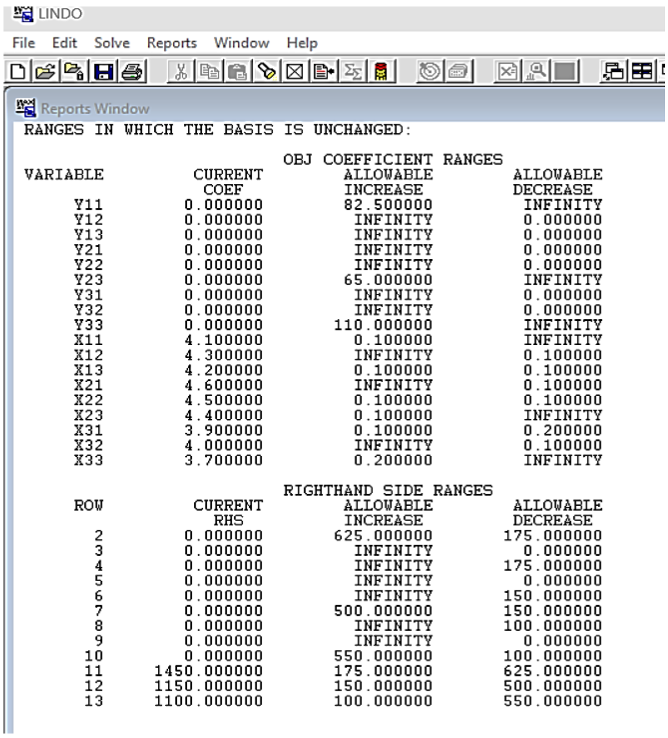
OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
Y11	350.000000	0.000000	INFINITY
Y12	405.000000	INFINITY	405.000000
Y13	375.000000	INFINITY	375.000000
Y21	430.000000	INFINITY	430.000000
Y22	360.000000	290.000000	INFINITY
Y23	400.000000	INFINITY	400.000000
Y31	380.000000	INFINITY	380.000000
Y32	360.000000	INFINITY	360.000000
Y33	350.000000	200.000000	INFINITY
X11	17.000000	1.000000	0.000000
X12	18.000000	INFINITY	1.000000
X13	17.000000	0.000000	INFINITY
X21	21.000000	0.000000	INFINITY
X22	20.000000	1.000000	INFINITY
X23	21.000000	INFINITY	0.000000
X31	18.000000	1.000000	1.000000
X32	19.000000	INFINITY	1.000000
X33	17.000000	1.000000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	0.000000	INFINITY	175.000000
3	0.000000	INFINITY	0.000000
4	0.000000	650.000000	175.000000
5	0.000000	500.000000	0.000000
6	0.000000	500.000000	150.000000
7	0.000000	INFINITY	150.000000
8	0.000000	INFINITY	100.000000
9	0.000000	INFINITY	0.000000
10	0.000000	550.000000	100.000000
11	1450.000000	175.000000	650.000000
12	1150.000000	150.000000	500.000000
13	1100.000000	100.000000	550.000000

Fig. 4. Allowable ranges in which the optimal solution for economic objective(Min z1) is unchanged.



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RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
Y11	0.000000	82.500000	INFINITY
Y12	0.000000	INFINITY	0.000000
Y13	0.000000	INFINITY	0.000000
Y21	0.000000	INFINITY	0.000000
Y22	0.000000	INFINITY	0.000000
Y23	0.000000	65.000000	INFINITY
Y31	0.000000	INFINITY	0.000000
Y32	0.000000	INFINITY	0.000000
Y33	0.000000	110.000000	INFINITY
X11	4.100000	0.100000	INFINITY
X12	4.300000	INFINITY	0.100000
X13	4.200000	0.100000	0.100000
X21	4.600000	INFINITY	0.100000
X22	4.500000	0.100000	0.100000
X23	4.400000	0.100000	INFINITY
X31	3.900000	0.100000	0.200000
X32	4.000000	INFINITY	0.100000
X33	3.700000	0.200000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	0.000000	625.000000	175.000000
3	0.000000	INFINITY	0.000000
4	0.000000	INFINITY	175.000000
5	0.000000	INFINITY	0.000000
6	0.000000	INFINITY	150.000000
7	0.000000	500.000000	150.000000
8	0.000000	INFINITY	100.000000
9	0.000000	INFINITY	0.000000
10	0.000000	550.000000	100.000000
11	1450.000000	175.000000	625.000000
12	1150.000000	150.000000	500.000000
13	1100.000000	100.000000	550.000000

Fig. 5. Allowable ranges in which the optimal solution for environmental objective (Min z2) is unchanged.

V. Conclusions

The study found an optimal solution for the product mix plan of the case study company. The single function value of this multi-objective optimization problem was computed and solution 2 is the best option for the product mix planning schedule of the case study company. The proposed mathematical model resulted in a pareto optimal solution for the achievement of the two objectives which are to minimize production costs and, at the same time minimize production wastes. The feasibility of the model and its effectiveness in achieving the goals of the company are demonstrated in the formulated model. A sensitivity analysis was also conducted in the study by applying variation of weights of the objective functions. This resulted in different decisions for the best product mix plan. It was found out that solution 2 is robust than solution 1. Moreover, the paper also analyzed the allowable ranges for the coefficients of the decision variables and the coefficients for the constraints, wherein the optimal solutions remain unchanged. Hence, this mixed integer linear model considers the variability of the values of the coefficients of the variables.

Many organizations can save huge amount of money and at the same time reduced waste by formulating a multi-objective optimization model. Solutions of this problem can be done in linear programming software but with additional steps of transforming the multi objectives function into single objective function. The common linear programming problem of product mix plan is no longer valid if it will set as having only one objective. Because in real settings of an enterprise, there exists several objectives to be met. This study could add to the knowledge of adapting a more sustainable production.

VI. Recommendations

The study recommends to have a more user-friendly template or model for the operations managers to utilize in their decisions about their product mix plan.

Conflict of Interest

“The authors declare no conflict of interest”.

Author Contributions

Gutierrez, M. T. conducted the research, analyzed the paper and wrote the paper; Ermita, P. suggested the literature review, and suggested some areas in the methodology. All authors approved the final version.

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