Parametrical Regulation of Economic Growth Based on the Dynamic Stochastic General Equilibrium Model of Frank Smets and Rafael Wouters

Abdykappar A. Ashimov, Yuriy V. Borovskiy, Rakhman A. Alshanov, Bakhyt T. Sultanov, and Mukhit A. Onalbekov

***Abstract*—The paper presents data on the estimation of parameters of dynamic stochastic general equilibrium model (DSGE model) of Frank Smets and Rafael Wouters based on the statistics of the Republic of Kazakhstan economy. Within the framework of economic growth problem on the basis of linear approximation of F. Smets and R. Wouters DSGE model of national economy the paper formulates the problem of maximization of expected discounted sum of GDP on the 10 years horizon with corresponding constraints imposed on interest rate, inflation and government spending. The considered problem was solved applying the parametrical regulation theory. The obtained results of solution to the stated problem have shown significant increase of the average GDP in the considered period compared with the corresponding basic forecast (without parametrical regulation).**

***Index Terms*—DSGE model, forecasting, parameter estimation, parametrical regulation.**

I. Introduction

It is well-known fact that simulation of economic growth is one of the most important directions of a state's economic policy. Within the framework of a rational macroeconomic policy in the sphere of economic growth it is required to have estimates of economic instruments values in order to ensure the desired growth of the national economy, under which we achieve such economic development, when the volumes of demand and supply, increasing from one period to another, are always equal to each other in the macro markets, capital and labor being fully utilized (dynamic balance). The above mentioned is in some way a requirement for mathematical models applicable to estimate the rational values of economic instruments of economic growth state policy. Dynamic stochastic general equilibrium models (DSGE models) [1] satisfy the requirements specified above.

The set of works [2, 3] considers the problems of economic growth based on the DSGE models as corresponding optimization problems, solutions to which are searched from the first-order conditions of the considered problems of the calculus of variation. This approach of finding optimal solutions involves complex transformations.

In this paper another approach based on the parametrical regulation theory [4] was firstly applied to solving the problem of economic growth on the basis of the DSGE model. In recent years the applied methods of the parametrical regulation theory have shown their efficiency on the set of applications based on the set of macroeconomic models, but they still are not used within the framework of the DSGE models.

This paper presents the results of estimating of parameters and characteristics of the shocks of DSGE model of Frank Smets and Rafael Wouters (Smets-Wouters model) [5] using the statistical data of the Republic of Kazakhstan economy. Also the results of the solution to problem of parametrical regulation of economic growth based on the linear approximation of the Smets-Wouters DSGE model are presented.

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The authors are with the Kazakh National Technical University named after K.I. Satpayev, Satpayev street 22, Almaty, 050013, Kazakhstan (e-mail: yuborovskiy@gmail.com, mukhon@list.ru).

II. Parameter Estimation of the Smets-Wouters DSGE Model Based on Statistical Data of the Republic of Kazakhstan Economy

In [5] the DSGE model was developed on the base of a given composition and behavior of agents and their interactions in a stochastic environment and adoption of rational expectations principles. Logarithmic linearization of the Smets-Wouters model around its steady state gives a linear model in the form:

$$A^{\theta}\hat{X}_{t-1} + B^{\theta}\hat{X}_t + C^{\theta}E_t\hat{X}_{t+1} + D^{\theta}H_t^{\Sigma_H} = 0 \qquad (1)$$

Here, E_t is mathematical expectation conditional on all information available at time t $(t = 1,2,3,\dots)$; $\hat{X}_t$ is a vector consisting of endogenous variables of the model ($\hat{X}_0$ is given); $H_t^{\Sigma_H}$ is a vector consisting of white Gaussian noises with zero mathematical expectations; Σ_H is a set of parameters consisting of the Gaussian noises $H_t^{\Sigma_H}$; $A^{\theta}, B^{\theta}, C^{\theta}, D^{\theta}$ are matrices of appropriate dimensions; θ is a set of parameters consisting of the model's structural parameters and the parameters of autoregressive shocks.

In this paper we consider the case in which the national economic policy is implemented applying the Taylor rule [6], which describes the behavior of the National Bank in setting interest rates, as well as rules for determining the amount of government spending.

Within the framework of the linear model (1) the Taylor rule is presented as:

$$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)\Big(\bar{\pi}_t + r_{\pi}(\hat{\pi}_{t-1} - \bar{\pi}_t) + rYYt{-}Ytp{+}r\Delta\pi\pi t{-}\pi t{-}1{+}r\Delta YYt{-}Ytp{-}Yt{-}1{-}Yt{-}1p{+}\eta_t^R \qquad (2)$$

and the rule of government spending is presented as:

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \eta_t^G \quad (3)$$

Here $\hat{R}_t, \hat{Y}_t, \hat{Y}_t^p, \hat{G}_t, \hat{\pi}_t$ are variables corresponding to yield on government bonds (1 + interest rate), GDP, potential GDP, government spending and inflation. η_t^R is interest rate shock, given as the white Gaussian noise; $\bar{\pi}_t$ is shock to inflation; η_t^G is government spending shock. ρ, r_π, $r_{\Delta\pi}$, $r_{\Delta Y}$, ρ_G are equations parameters.

The solution to the linear model (1) on base of the Blanchard-Khan algorithm [7, 8] is written as the first-order vector autoregressive model:

$$\hat{X}_t = Q^\theta \hat{X}_{t-1} + F^\theta H_t^{\Sigma H}, t = 1,2,3, \ldots \quad (4)$$

where $\hat{X}_0$ is given. Q^θ, F^θ are matrices of appropriate dimensions.

Estimation of the parameters of the model (1) was obtained applying the Bayesian estimation method (using the Metropolis-Hastings algorithm with 4000000 simulations) via Kalman filter [9]. We used quarterly data from 2002Q1 to 2011Q3 of seven macroeconomic indicators of the Republic of Kazakhstan including real GDP, real investment, real consumption, employment, average real wage, refinancing rate of the National bank and inflation. Linearly detrended logarithms of statistical data were used as observation to model (4). The values of estimated parameters described in [5] are presented in Table I.

TABLE I: PARAMETER ESTIMATES OF THE SMETS-WOUTERS MODEL.

Parameter	Estimated value	Parameter	Estimated value
α	0.3125	ρ	0.8967
β	0.9800	r_Y	0.1198
τ	0.0250	$r_{\Delta Y}$	0.0069
C_Y	0.5700	ρ_A	0.4044
I_Y	0.2600	ρ_B	0.4001
λ_W	0.5000	ρ_G	0.2975
$1/\varphi$	3.9297	ρ_L	0.7262
σ_C	0.8738	ρ_I	0.6311
h	0.2558	ρ_π	0.6731
ξ_W	0.9256	$st.d.\ \eta_t^A$	1.9318
σ_L	1.8452	$st.d.\ \eta_t^B$	3.7493
ξ_P	0.8670	$st.d.\ \eta_t^G$	2.6658
ξ_E	0.3781	$st.d.\ \eta_t^L$	3.1822
γ_W	0.3784	$st.d.\ \eta_t^I$	0.2260
γ_P	0.3309	$st.d.\ \eta_t^R$	0.0825
ψ''/ψ'	0.4748	$st.d.\ \eta_t^\pi$	0.0502
ϕ	2.6152	$st.d.\ \eta_t^Q$	7.9159
r_π	0.8579	$st.d.\ \eta_t^P$	0.9303
$r_{\Delta\pi}$	0.0116	$st.d.\ \eta_t^W$	1.4088

The quality of the method used to find the parameter estimates was tested via retroforecasting. For this purpose we forecasted observable economic indicators for 4 periods from 2010Q4 to 2011Q3. Deviations of the obtained forecasted values of economic indicators from the corresponding statistical date were about 3%.

III. PARAMETRICAL REGULATION IN THE SPHERE OF STATE POLICY FOR ECONOMIC GROWTH BASED ON THE SMETS-WOUTERS

The additive terms η_t^R, η_t^G in the expressions (2), (3) were selected as the instruments for state policy implementation in the sphere of economic growth. Their desired values were sought in the form of deterministic values replacing the corresponding shocks according the follow mathematical formulation of the parametrical regulation problem.

On the basis of the model (4) find values of the instruments $\eta_{T+1}^R, \ldots, \eta_{T+40}^R, \eta_{T+1}^G, \ldots, \eta_{T+40}^G$ that provide maximum to the criterion:

$$K(\eta_{T+1}^R, \ldots, \eta_{T+40}^R, \eta_{T+1}^G, \ldots, \eta_{T+40}^G) = E_T \sum_{i=1}^{40} \beta^i Y_{T+i} \quad (5)$$

under additional constraints imposed on the following endogenous variables of the model: forecasts of inflation and interest rate on bonds during this period should not deviate from their steady state values by more 0.5%, and the amount of government spending by more than 5.0% from its steady state value. Here T is the number of quarter that corresponds to 2011Q3; Y_{T+i} is real GDP of the period $[T+i-1,\ T+i]$; β is discount factor.

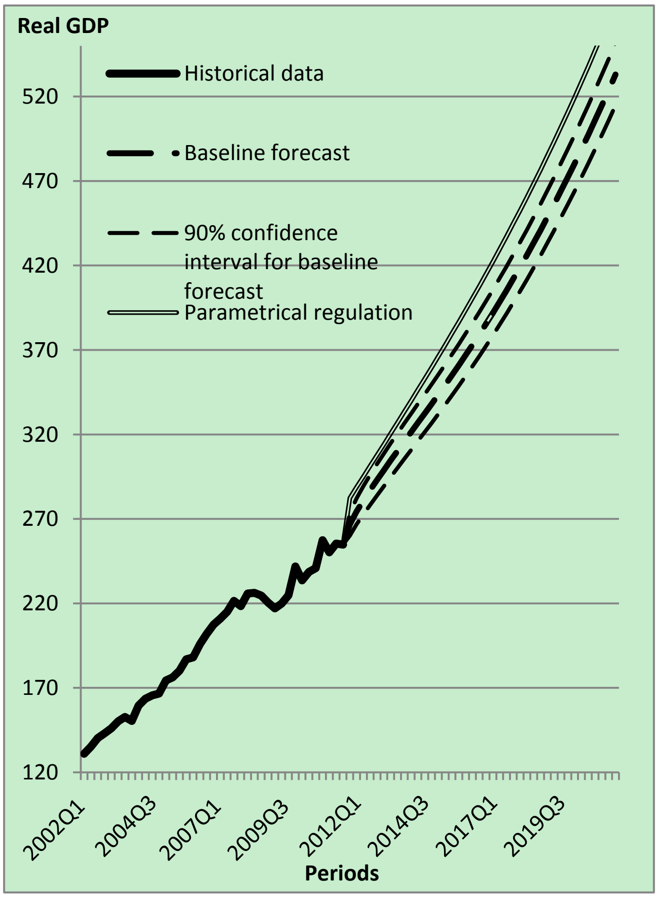

Fig. 1. Gross domestic product (in billions of tenge, in average prices of 1994 year).

This optimization problem belongs to the class of problems of the calculus of variation. Parametrical regulation of economic growth provided 6% growth of average GDP on the considered interval compared with the forecast under the base scenario obtained applying model (4) (See Fig. 1).

IV. Summary

1) The parameters of the Smets-Wouters DSGE model have been estimated for economy of the Republic of Kazakhstan.
2) The problem of parametrical regulation of economic growth based on the considered model was formulated and solved.
3) The obtained results can be used for designing and implementing effective public policy in the sphere of economic growth.

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Abdykappar A. Ashimov is founder of the national research school in the theory of control systems and technical cybernetics. He is the academician of the National Academy of Sciences of the Republic of Kazakhstan and the International Academy of Informatization, doctor of technical sciences, professor. His research interests are systems analysis, applied theory of dynamical systems and control theory, theory of parametrical regulation of nonlinear dynamical systems evolution. He is the author of over 400 scientific papers. Abdykappar A. Ashimovwas awarded the medal "For Valiant Labor"(1970), the title of "Honored Scientist of the Kazakh SSR"(1980),the order of "People's Friendship"(1981),the diploma of the Presidium of the Supreme Soviet of the Kazakh SSR(1984),the prize of the Council Ministers of the Kazakh SSR in the field of science and technology(1989),the order "Parasat"(2006), the award "The Outstanding Paper" of Kybernetesjournal for the best paper presented at the 15 International Congress on Cybernetics and Systems (2011) .