Analysis of the Dynamics in the Relationships between the Consumption of Various Types of Fresh Meat by Japanese Households

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***Abstract*—What kinds of relationships exist among the growth rates of consumption of beef, pork, and chicken in Japan? This paper studies the dynamics of fresh meat consumption using monthly data from January 1990 to March 2014 from Japan's Agriculture & Livestock Industries Corporation. First, a time-varying coefficient vector autoregressive model is constructed for a time series of fresh meat consumption, and its parameters are estimated using a Bayesian method. The time-varying power contribution and time-varying covariance function for the time series are then obtained based on the estimates for the model. The results show that the power contribution to the consumption of chicken from that of beef is very significant. The consumption of beef gives a stronger power contribution to that of pork at lower frequencies. Furthermore, the consumption of pork gives a stronger power contribution to that of chicken at lower and higher frequencies, and especially the latter in recent years.**

***Index Terms*—Bayesian modeling, fresh meat consumption, time-varying coefficient, VAR model.**

I. INTRODUCTION

In this paper, we focus on the consumption of various types of fresh meat in Japan, namely beef, pork, and chicken. When the consumption of one type of meat fluctuates as the result of a shock, the consumption of other kinds of meat will be affected. For instance, when beef consumption drops significantly following a bovine spongiform encephalopathy outbreak, the consumption of pork and chicken increases. The aim of this paper is to analyze the relationships among changes in the consumption of beef, pork, and chicken in Japan using a time-varying coefficient vector autoregressive (TVCVAR) model.

Major studies on the interactions between various types of meat consumption or price include those of Chang and Griffith [1], Andersen *et al*. [2], and Hajko and Jaroslav [3]. Chang and Griffith [1] analyzed the relationships between Australian beef prices at farm, wholesale, and retail levels using VAR models and found that all three prices were cointegrated. Furthermore, the wholesale price was found to be weakly exogenous. The latter result might be an indication of market inefficiency that is due, in part, to price leveling, which is often practiced in the beef marketing system. Andersen *et al*. [2] analyzed Danish dynamic meat price and quantity transmissions using a VAR model of market-clearing quantities and prices from the Danish pork, chicken, and beef markets. Their main result implied that pork, chicken, and beef are close substitutes. Hajko and Jaroslav [3] focused on the relevant markets for various types of meat in the Czech Republic. Based on a cointegration analysis and testing of Granger causality using a VAR model, they obtained the following results: for chicken, the market in the Czech Republic can be considered independent, both geographically and by product. For pork, the relevant market includes Germany and Slovakia in addition to the Czech Republic. The relevant beef market includes the Czech Republic and Germany.

A vector autoregressive (VAR) modeling approach is useful for analyzing the relationships within vector time series [4]. However, a VAR model can only be applied to stationary time series. Moreover, in a conventional VAR modeling approach, coefficients in the models are treated as constant parameters despite the use of long-term time series data. Although this reflects the assumption of invariability in the model structure, the assumption that there are no structural changes whatsoever when the model covers a period of several decades is clearly unrealistic. Thus, much of the previous research is not based on an appropriate dynamic framework.

Jiang and Kitagawa proposed an approach for vector time series with nonstationary covariance by developing a time-varying coefficient vector autoregressive (TVCVAR) modeling method [5]. The TVCVAR model can be used to explain the dynamic relationship between all variates in a vector time series (see [6]). For example, Kyo and Noda [7] apply the Bayesian TVCVAR modeling approach proposed by Jiang and Kitagawa [5]. Specifically, Kyo and Noda [7] consider the VAR model with time-varying coefficients that treats both oil price fluctuations and industrial production growth as endogenous variables. Regarding the influence that an oil price change has on an economy, the first and second oil shocks tend to be the focus. That is, the recognition of past oil shock events is dominant. However, oil prices have been fluctuating in recent years, and in particular have had an influence on industrial production since the 2000s that cannot be disregarded. In this paper, the TVCVAR modeling approach is applied to the analysis of the dynamic relationship among consumption growth rates for beef, pork, and chicken in Japan.

The rest of this paper is organized as follows. In Section II, we present our Bayesian TVCVAR modeling and the

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procedure for parameter estimation. In Section III, we show the results and discuss some implications. Section IV concludes the paper.

II. Modeling and Estimation Scheme

A. Model Construction

Consider a set of monthly data as the seasonally adjusted time series x_{n1}, x_{n2}, and x_{n3}, which expresses the consumption of beef, pork, and chicken, respectively, with n denoting the month. The 3-month-ahead growth rates can then be calculated as

$$y_{ni} = \frac{x_{ni} - x_{(n-3)i}}{x_{(n-3)i}} \quad (i = 1, 2, 3).$$

We regard $\boldsymbol{y}_n = (y_{n1}, y_{n2}, y_{n3})^{\mathrm{T}}$ as a 3-variate time series, and introduce the following TVCVAR model:

$$\boldsymbol{y}_n = \textstyle\sum_{\ell=1}^{L} \boldsymbol{A}_\ell(n)\, \boldsymbol{y}_{n-\ell} + \boldsymbol{u}_n, \tag{1}$$

where L is the model order, and $\boldsymbol{A}_\ell(n)$ $(\ell = 1, 2, \cdots, L)$ are time-varying coefficient matrices for each lag at time n. In Eq. (1), $\boldsymbol{u}_n$ is a 3-variate Gaussian white noise sequence with zero mean and covariance matrix $\boldsymbol{\Sigma}(n)$. It is assumed that $\boldsymbol{u}_n$ and $\boldsymbol{y}_{n-\ell}$ are independent of each other for $\ell > 0$.

To estimate the parameters efficiently, we construct a TVCVAR model with a simultaneous response as

$$\boldsymbol{y}_n = \textstyle\sum_{\ell=1}^{L} \boldsymbol{B}_\ell(n)\, \boldsymbol{y}_{n-\ell} + \boldsymbol{w}_n, \tag{2}$$

where $\boldsymbol{w}_n = (w_{n1}, w_{n2}, w_{n3})^{\mathrm{T}}$ is a 3-variate Gaussian white noise sequence with zero mean and covariance matrix $\boldsymbol{W} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2)$. The coefficient matrices $\boldsymbol{B}_\ell(n)$ $(\ell = 0, 1, \cdots, L)$ are defined as follows:

$$\boldsymbol{B}_0(n) = \begin{bmatrix} 0 & 0 & 0 \\ b_{210}(n) & 0 & 0 \\ b_{310}(n) & b_{320}(n) & 0 \end{bmatrix},$$

$$\boldsymbol{B}_\ell(n) = \begin{bmatrix} b_{11\ell}(n) & b_{12\ell}(n) & b_{13\ell}(n) \\ b_{21\ell}(n) & b_{22\ell}(n) & b_{23\ell}(n) \\ b_{31\ell}(n) & b_{32\ell}(n) & b_{33\ell}(n) \end{bmatrix}$$
$$(\ell = 1, 2, \cdots, L).$$

In particular, $\boldsymbol{B}_0(n)$ is called a simultaneous response matrix. For each element of $\boldsymbol{y}_n$, the model in Eq. (2) can be rewritten as follows:

$$y_{n1} = \sum_{j=1}^{3} \sum_{\ell=1}^{L} b_{1j\ell}(n) y_{(n-\ell)j} + w_{n1},$$
$$w_{n1} \sim N(0, \sigma_1^2), \tag{3}$$

$$y_{n2} = b_{210}(n) y_{n1} + \sum_{j=1}^{3} \sum_{\ell=1}^{L} b_{2j\ell}(n) y_{(n-\ell)j} + w_{n2},$$
$$w_{n2} \sim N(0, \sigma_2^2), \tag{4}$$

$$y_{n2} = \sum_{j=1}^{2} b_{3j0}(n)\, y_{nj} + \sum_{j=1}^{3} \sum_{\ell=1}^{L} b_{2j\ell}(n) y_{(n-\ell)j} + w_{n2},$$
$$w_{n3} \sim N(0, \sigma_3^2). \tag{5}$$

It is assumed that w_{n1}, w_{n2}, and w_{n3} are independent of each other. Thus, we can estimate the parameters for each model separately in Eqs. (3), (4), and (5), so the efficiency of parameter estimation can be improved. This is the first advantage of using a form of the TVCVAR model with a simultaneous response.

To estimate the time-varying coefficients, we apply a Bayesian method using smoothness priors of order 1 for the nonzero elements in the matrices $\boldsymbol{B}_\ell(n)$ $(\ell = 0, 1, \cdots, L)$. That is, we introduce a set of smoothness priors of order 1 in the form

$$b_{ij\ell}(n) - b_{ij\ell}(n-1) = v_{ij\ell}(n), \tag{6}$$

where $v_{ij\ell}(n)$ is a Gaussian white noise sequence with zero mean and unknown variance τ_i^2.

It can be confirmed that the models in Eqs. (1) and (2) are linked by the relationships

$$\boldsymbol{A}_\ell(n) = \left(\boldsymbol{I} - \boldsymbol{B}_0(n)\right)^{-1} \boldsymbol{B}_\ell(n) \quad (\ell = 1, 2, \cdots, L), \tag{7}$$

$$\boldsymbol{\Sigma}(n) = \left(\boldsymbol{I} - \boldsymbol{B}_0(n)\right)^{-1} \boldsymbol{W} \left(\boldsymbol{I} - \boldsymbol{B}_0(n)\right)^{-\mathrm{T}}. \tag{8}$$

Therefore, if the parameters in the model in Eq. (2) are given, those in the model in Eq. (1) can be obtained using Eqs. (7) and (8).

B. Estimation of Time-Varying Coefficients

Now, we set

$$\boldsymbol{x}_n^{(1)} = (b_{111}(n) \quad b_{121}(n) \quad b_{131}(n) \quad \cdots b_{11L}(n) \quad b_{12L}(n) \quad b_{13L}(n)),$$

$$\boldsymbol{x}_n^{(2)} = (b_{210}(n) \quad b_{211}(n) \quad b_{221}(n) \quad b_{231}(n) \quad \cdots b_{21L}(n) \quad b_{22L}(n) \quad b_{23L}(n)),$$

$$\boldsymbol{x}_n^{(3)} = (b_{310}(n) \quad b_{320}(n) \quad b_{311}(n) \quad b_{321}(n) \quad b_{331}(n) \quad \cdots b_{31L}(n) \quad b_{32L}(n) \quad b_{33L}(n)),$$

$$\boldsymbol{v}_n^{(1)} = (v_{111}(n) \quad v_{121}(n) \quad v_{131}(n) \quad \cdots v_{11L}(n) \quad v_{12L}(n) \quad v_{13L}(n)),$$

$$\boldsymbol{v}_n^{(2)} = (v_{210}(n) \quad v_{211}(n) \quad v_{221}(n) \quad v_{231}(n) \quad \cdots v_{21L}(n) \quad v_{22L}(n) \quad v_{23L}(n)),$$

$$\boldsymbol{v}_n^{(3)} = (v_{310}(n) \quad v_{320}(n) \quad v_{311}(n) \quad v_{321}(n) \quad v_{331}(n) \quad \cdots v_{31L}(n) \quad v_{32L}(n) \quad v_{33L}(n))$$

$$\boldsymbol{H}_{n1} = (y_{(n-1)1} \quad y_{(n-1)2} \quad y_{(n-1)3} \quad \cdots y_{(n-L)1} \quad y_{(n-L)2} \quad y_{(n-L)3}),$$

$$\boldsymbol{H}_{n2} = (y_{n1} \quad y_{(n-1)1} \quad y_{(n-1)2} \quad y_{(n-1)3} \quad \cdots y_{(n-L)1} \quad y_{(n-L)2} \quad y_{(n-L)3}),$$

$$\boldsymbol{H}_{n3} = (y_{n1} \quad y_{n2} \quad y_{(n-1)1} \quad y_{(n-1)2} \quad y_{(n-1)3} \quad \cdots y_{(n-L)1} \quad y_{(n-L)2} \quad y_{(n-L)3})$$

and

$$\boldsymbol{G}_i = \boldsymbol{F}_i = \boldsymbol{I}, \qquad \boldsymbol{Q}_i = \tau_i^2 \boldsymbol{I}, \qquad R_{\mathrm{i}} = \sigma_i^2 \quad (i = 1, 2, 3)$$

with $\boldsymbol{I}$ being an identity matrix of size $3L + i - 1$. Then, one of the models in Eqs. (3)–(5) together with Eq. (6) can be expressed by the following state space model:

$$\boldsymbol{x}_n^{(i)} = \boldsymbol{F}_i \boldsymbol{x}_{n-1}^{(i)} + \boldsymbol{G}_i \boldsymbol{v}_n^{(i)}, \tag{9}$$

$$\boldsymbol{y}_{ni} = \boldsymbol{H}_{ni} \boldsymbol{x}_n^{(i)} + \boldsymbol{w}_{ni}, \tag{10}$$

with respect to $i = 1, 2, 3$, respectively. In the state space model comprising Eqs. (9) and (10), the time-varying coefficients are included in the state vector $\boldsymbol{x}_n^{(i)}$, so their estimates can be obtained from the estimate of $\boldsymbol{x}_n^{(i)}$. Moreover, the parameters, σ_i^2 and τ_i^2, which are called hyperparameters, can be estimated using the maximum likelihood method.

Let $\boldsymbol{x}_0^{(i)}$ denote the initial value of the state $\boldsymbol{x}_n^{(i)}$ and Y_m denote a set of observations of the time series $\boldsymbol{y}_n$ up to the time point m. Assume that $\boldsymbol{x}_0^{(i)} \sim \mathrm{N}\left(\boldsymbol{x}_{0|0}^{(i)}, \boldsymbol{V}_{0|0}^{(i)}\right)$. It is well known that the distribution $f\left(\boldsymbol{x}_n^{(i)}|Y_m\right)$ for the state $\boldsymbol{x}_n^{(i)}$ conditional on Y_m is Gaussian, so it is only necessary to obtain the mean $\boldsymbol{x}_n^{(i)}$ and the covariance matrix $\boldsymbol{V}_{n|m}^{(i)}$ of $\boldsymbol{x}_n^{(i)}$ with respect to $f\left(\boldsymbol{x}_n^{(i)}|\, Y_m\right)$.

When the values of L, σ_i^2, and τ_i^2, the initial distribution $\mathrm{N}\left(\boldsymbol{x}_{0|0}^{(i)}, \boldsymbol{V}_{0|0}^{(i)}\right)$, and a set of observations up to the period N are given, the estimates for the state $\boldsymbol{x}_n^{(i)}$ can be obtained using the well-known Kalman filter (for $n = 1, 2, \cdots, N$) and fixed-interval smoothing (for $n = N-1, N-2, \cdots, 1$) recursively as follows (see, for example, [8] and [9]):

[Kalman filter (step 1): One-step-ahead prediction]

$$\boldsymbol{x}_{n|n-1}^{(i)} = \boldsymbol{F}_i \boldsymbol{x}_{n-1|n-1}^{(i)},$$

$$\boldsymbol{V}_{n|n-1}^{(i)} = \boldsymbol{F}_i \boldsymbol{V}_{n-1|n-1}^{(i)} \boldsymbol{F}_i^{\mathrm{T}} + \boldsymbol{G}_i \boldsymbol{Q}_i \boldsymbol{G}_i^{\mathrm{T}}.$$

[Kalman filter (step 2): Filter]

$$\boldsymbol{K}_{ni} = \boldsymbol{V}_{n|n-1}^{(i)} \boldsymbol{H}_{ni}^{\mathrm{T}} \left(\boldsymbol{H}_{ni} \boldsymbol{V}_{n|n-1}^{(i)} \boldsymbol{H}_{ni}^{\mathrm{T}} + R_i\right)^{-1},$$

$$\boldsymbol{x}_{n|n}^{(i)} = \boldsymbol{x}_{n|n-1}^{(i)} + \boldsymbol{K}_{ni}\left(\boldsymbol{y}_{ni} - \boldsymbol{H}_{ni} \boldsymbol{x}_{n|n-1}^{(i)}\right),$$

$$\boldsymbol{V}_{n|n}^{(i)} = (\boldsymbol{I} - \boldsymbol{K}_{ni} \boldsymbol{H}_{ni}) \boldsymbol{V}_{n|n-1}^{(i)}.$$

[Fixed-interval smoothing]

$$\boldsymbol{D}_{ni} = \boldsymbol{V}_{n|n}^{(i)} \boldsymbol{F}_i^{\mathrm{T}} \left(\boldsymbol{V}_{n+1|n}^{(i)}\right)^{-1},$$

$$\boldsymbol{x}_{n|N}^{(i)} = \boldsymbol{x}_{n|n}^{(i)} + \boldsymbol{D}_{ni}\left(\boldsymbol{x}_{n+1|N}^{(i)} - \boldsymbol{x}_{n+1|n}^{(i)}\right),$$

$$\boldsymbol{V}_{n|N}^{(i)} = \boldsymbol{V}_{n|n}^{(i)} + \boldsymbol{D}_{ni}\left(\boldsymbol{V}_{n+1|N}^{(i)} - \boldsymbol{V}_{n+1|n}^{(i)}\right) \boldsymbol{D}_{ni}^{\mathrm{T}}.$$

Then, the posterior distribution of $\boldsymbol{x}_n^{(i)}$ can be given by $\mathrm{N}\left(\boldsymbol{x}_{n|N}^{(i)}, \boldsymbol{V}_{n|N}^{(i)}\right)$, and subsequently the estimates for the time-varying coefficients can be obtained because the state space model described by Eqs. (9) and (10) incorporates the coefficients in the state vector $\boldsymbol{x}_n^{(i)}$. Furthermore, the time-varying cross-spectrum, time-varying power contribution, and time-varying covariance function can be obtained based on the estimates of $\boldsymbol{A}_\ell(n)$ and $\boldsymbol{\Sigma}(n)$ using the method proposed in [4]. In particular, the time-varying power contribution and time-varying covariance function help to explain the dynamic relationship between every variate in the vector time series.

C. Estimation of Constant Parameters

Let $Y_m = \{y_1, y_2, \cdots, y_m\}$ be the set of observations for the time series y_n up to the time point m, with Y_0 being an empty set. When the value of model order L and the whole of the time series data Y_N are given, the likelihood function of the hyperparameters σ_i^2 and τ_i^2 $(i = 1, 2, 3)$ is defined approximately by

$$\begin{aligned} f(Y_N|\sigma_1^2, \tau_1^2, \sigma_2^2, \tau_2^2, \sigma_3^2, \tau_3^2) &= \prod_{n=1}^{N} f_n^{(1)}\left(y_{n1}|Y_{(n-1)}; \sigma_1^2, \tau_1^2\right) f_n^{(2)}\left(y_{n2}|y_{n1}, Y_{(n-1)}; \sigma_2^2, \tau_2^2\right) \\ &\quad \times f_n^{(3)}\left(y_{n3}|y_{n1}, y_{n2}, Y_{(n-1)}; \sigma_3^2, \tau_3^2\right), \end{aligned}$$

where $f_n^{(1)}\left(y_{n1}|Y_{(n-1)}; \sigma_1^2, \tau_1^2\right)$ is the conditional density of y_{n1} given the past observations $Y_{(n-1)}$ together with the values of σ_1^2 and τ_1^2, and so on. As given by [9], using the Kalman filter means that conditional densities are normal densities given by

$$f_n^{(1)}\left(y_{n1}|Y_{(n-1)}; \sigma_1^2, \tau_1^2\right) = \frac{1}{\sqrt{2\pi s_{n|n-1}^{(1)}}} \exp\left\{-\frac{\left(y_{n1} - \hat{y}_{n|n-1}^{(1)}\right)^2}{2 s_{n|n-1}^{(1)}}\right\},$$

$$f_n^{(2)}\left(y_{n2}|y_{n1}, Y_{(n-1)}; \sigma_2^2, \tau_2^2\right) = \frac{1}{\sqrt{2\pi s_{n|n-1}^{(2)}}} \exp\left\{-\frac{\left(y_{n2} - \hat{y}_{n|n-1}^{(2)}\right)^2}{2 s_{n|n-1}^{(2)}}\right\},$$

$$f_n^{(3)}\left(y_{n3}|y_{n1}, y_{n2}, Y_{(n-1)}; \sigma_3^2, \tau_3^2\right) = \frac{1}{\sqrt{2\pi s_{n|n-1}^{(3)}}} \exp\left\{-\frac{\left(y_{n3} - \hat{y}_{n|n-1}^{(3)}\right)^2}{2 s_{n|n-1}^{(3)}}\right\},$$

where $\hat{y}_{n|n-1}^{(i)}$ is the mean for the one-step-ahead prediction of y_{ni} and $s_{n|n-1}^{(i)}$ is the variance of the predictive error, which are respectively given by

$$\hat{y}_{n|n-1}^{(i)} = \boldsymbol{H}_{ni} \boldsymbol{x}_{n|n-1}^{(i)},$$

$$s_{n|n-1}^{(i)} = \boldsymbol{H}_{ni} \boldsymbol{V}_{n|n-1}^{(i)} \boldsymbol{H}_{ni}^{\mathrm{T}} + R_i \quad (i = 1, 2, 3).$$

By taking the logarithm of $f(Y_N|\sigma_1^2, \tau_1^2, \sigma_2^2, \tau_2^2, \sigma_3^2, \tau_3^2)$, the log-likelihood is given by

$$\begin{aligned} \ell(\sigma_1^2, \tau_1^2, \sigma_2^2, \tau_2^2, \sigma_3^2, \tau_3^2) &= \log f(Y_N|\sigma_1^2, \tau_1^2, \sigma_2^2, \tau_2^2, \sigma_3^2, \tau_3^2) \\ &= \ell_1(\sigma_1^2, \tau_1^2) + \ell_2(\sigma_2^2, \tau_2^2) + \ell_3(\sigma_3^2, \tau_3^2), \end{aligned}$$

where $\ell_1(\sigma_1^2,\tau_1^2)$, $\ell_2(\sigma_2^2,\tau_2^2)$, and $\ell_3(\sigma_3^2,\tau_3^2)$ are the partial log-likelihood functions, which are given by

$$\ell_1(\sigma_1^2,\tau_1^2)=\sum_{n=1}^{N}\log f_n^{(1)}\left(y_{n1}|Y_{(n-1)};\sigma_1^2,\tau_1^2\right),$$

$$\ell_2(\sigma_2^2,\tau_2^2)=\sum_{n=1}^{N}\log f_n^{(2)}\left(y_{n2}|y_{n1},Y_{(n-1)};\sigma_2^2,\tau_2^2\right),$$

$$\ell_3(\sigma_3^2,\tau_3^2)=\sum_{n=1}^{N}\log f_n^{(3)}\left(y_{n3}|y_{n1},y_{n2},Y_{(n-1)};\sigma_3^2,\tau_3^2\right).$$

Thus, the estimates of the hyperparameters can be obtained using the maximum likelihood method, i.e., the estimates for the hyperparameters σ_1^2,τ_1^2 are obtained by maximizing $\ell_1(\sigma_1^2,\tau_1^2)$, and so on for the other hyperparameters.

Theoretically, the value of model order L should be determined via the minimum AIC method (see [10]). However, we use a vague distribution to set $\mathrm{N}\left(\boldsymbol{x}_{0|0}^{(i)},\boldsymbol{V}_{0|0}^{(i)}\right)$ by $\boldsymbol{x}_{0|0}^{(i)}$ and $\boldsymbol{V}_{0|0}^{(i)}=\delta\boldsymbol{I}$ for $i=1,2,3$ with δ being a sufficiently large positive number. In this case, the values of L can also be determined by the maximum likelihood method because the number of hyperparameters in the model is identical for different values of L.

III. Results and Implications

We applied the proposed approach to the monthly statistics from January 1990 to March 2014, which we obtained from Japan's Agriculture & Livestock Industries Corporation.

Fig. 1 shows the line graphs of the time series data for y_{n1}, y_{n2}, and y_{n3}, which express the 3-month-ahead growth rates for the consumption of beef, pork, and chicken, respectively.

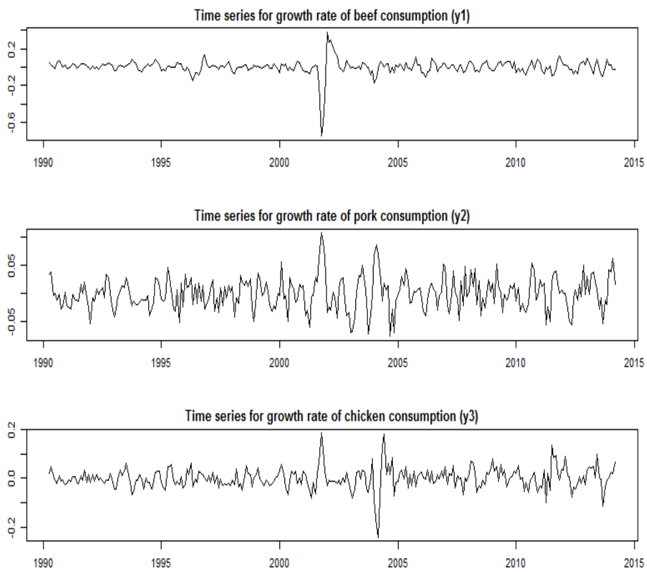

Fig. 1. Data for the 3-month-ahead time series.

First, we show the results for determining the values of L. Table I shows the maximum log-likelihood values for each value of model order L.

TABLE I: Log-Likelihood Values for the TVCVAR Model

$L=1$	$L=2$	$L=3$	$L=4$	$L=5$
1591.16	1585.09	1627.62	1612.52	1613.40
$L=6$	$L=7$	$L=8$	$L=9$	$L=10$
1629.52	1626.60	1621.98	1611.74	1599.56

It can be seen from Table I that the log-likelihood value is maximized when $L=6$, so we determined a value of the model order using $L=6$ based on the maximum likelihood method.

Fig. 2 and Fig. 3 show the time-varying power contribution and the time-varying cross-covariance, respectively.

In Fig. 2, the panels on the diagonal show the estimates for the time-varying power contribution for the same type of meat. For instance, the panel in the second row and the second column shows the time-varying power contribution to the pork consumption growth rate of the pork consumption growth rate. The other panels show the time-varying power contribution for different types of meats. For instance, the panel in the first row and the second column shows the time-varying power contribution of the pork consumption growth rate to the beef consumption growth rate.

A number of conclusions can be drawn from this result. Changes in the consumption of beef have not been strongly affected by changes in the consumption of pork and chicken. Similarly, changes in the consumption of pork have not been overly affected by changes in the consumption of beef and chicken. However, the consumption of beef has a long-term impact on that of chicken, and is also affected by its own past consumption. This latter feature can also be seen in relation to the consumption of pork and chicken. This can be interpreted as a kind of habit formation in the consumption of livestock products.

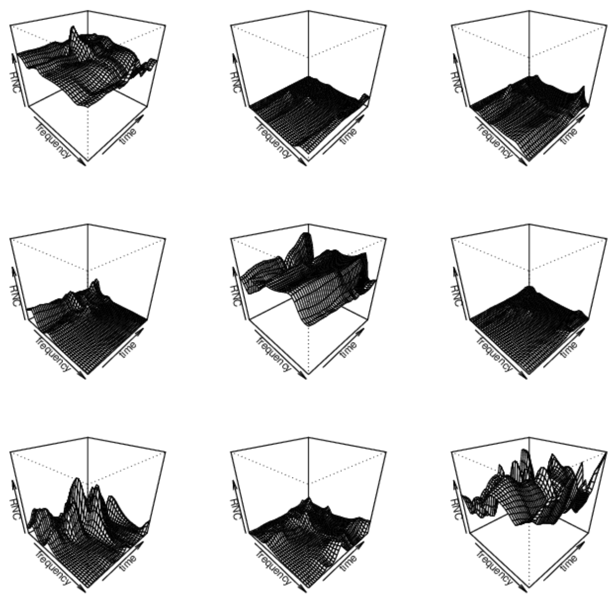

Fig. 2. Estimates of time-varying power contribution.

In Fig. 3, the diagonal panels show the estimates for time-varying cross-covariance for the same type of meat. The panel in the third row and the third column shows the time-varying power cross-covariance between the past chicken consumption growth rate and the present chicken consumption growth rate. The other panels show the time-varying cross-covariance for different types of meats.

For instance, the panel in the first row and the third column shows the time-varying cross-covariance between the beef consumption growth rate and the chicken consumption growth rate.

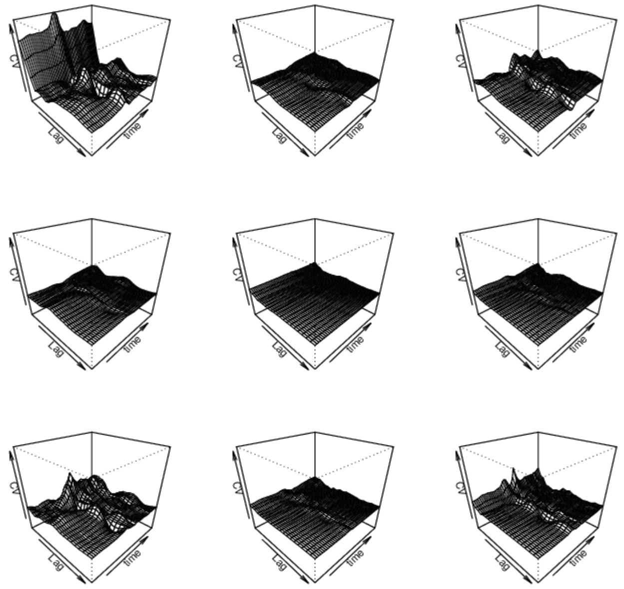

Fig. 3. Estimates of time-varying cross-covariance.

Fig. 3 shows that there is a clear relationship between the present and past consumption growth rates of beef, with a short lag time. The growth rates of past chicken consumption are related, in part, to the growth rates of present chicken consumption, with a short lag time. However, we cannot find a clear relationship between the present and past consumption growth rates of pork.

IV. Conclusions

Some interesting findings can be distilled from these results. No significant changes over time can be observed in the correlations between the changes in the consumption of beef and pork, the changes in the consumption of chicken and pork, and the changes in the present and past consumption of beef. However, there are changes over time in the correlations between the consumption of beef and chicken, the present and past consumption of chicken, and the present and past consumption of beef. Moreover, there is a correlation between the consumption of beef and chicken, but no clear correlation can be observed between the consumption of beef and pork.

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