A Multi-stage Manufacturing and Delivering Medical Model with Dynamic Decision-Making System Based on SIQR Model

Yihui Chen, Donglin Wang, Can Pu, and Wenxiao Mou

***Abstract*—Outbreak of Ebola has urged local government and medical institution to control the epidemic with low cost. We propose a multi-stage manufacturing and delivering medical model with dynamic decision-making system based on SIQR model, and then Linear Programming is employed to obtain the optimal distribution plan. In the model, the use of vaccine can quickly immunize people, while the treatment of drugs will last for a period of time. On the basis of the different mechanisms of vaccine and drugs, we improve the traditional SIQR model. The use of drugs and vaccine in each time period will have influence on the epidemic in the next time period, which will in turn influence production and delivery of drugs and vaccine. Considering the features of Ebola, we set up simulated data by experience. It turns out that the model can effectively simulate the spread and control of an epidemic. Therefore, the model in this paper is reasonable and is worthy of generalization and application.**

***Index Terms*—Epidemic control, dynamic decision-making, multi-stage, SIQR model.**

I. Introduction

Outbreaks of Ebola in the year 2014 in West Africa are associated with case fatality rates between 25 and 90 per cent. Control of outbreaks requires coordinated medical services. The three worst affected areas are Guinea, Liberia and Sierra Leone. The advent of Ebola has now sharply alarmed us that epidemic is always one of our cruelest enemies, which has been greatly threatening our lives and property [1].

To control the spread of Ebola, we should block the transmission on the one hand, and try to cure the infecting source on the other hand. The effect of treatment might be determined by factors such as the speed of manufacturing of vaccine or drug, quantity of the medicine needed, medicine delivery systems, and the location of medicine delivery, etc. It is thus in great need to set up a theoretical model containing both aspects. The parameters of the model can be optimized to effectively control the spread of the epidemic. The rationalization of the model can be verified by numerical data.

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II. Review of Literature

A. Model Based on Differential Equations

The research concerning epidemic prevention and control could be traced back to 1927 when Kermack and Mckendrick [2] proposed an epidemic model (*SIR model* for short). In *SIR* model, they divided people into three different classes: 1) *S* stands for susceptible people who can be infected by exposure to source of infection; 2) *I* stands for infected people who have already been infected and remain infectious. 3) *R* stands for people removed from the system who have been removed because of recovery and death of the disease. People immune of the disease through vaccinations can also be classified into this group. Based on their research on the transmission rules and epidemic trend of the disease, they proposed threshold theory. Once the number of susceptible people is higher than threshold, the epidemic will continue to exist. If the opposite happens, the epidemic goes extinct. The model is largely supported by data from many serious infections in human history.

In 2003, Lipsitch *et al.* published an article called *"Transmission dynamics and control of severe acute respiratory syndrome"* [3]. They built *SEIR model* based on the previous *SIR model*. *E* stands for people who stay in the incubation period. Then they analyzed the influence on the spread of the epidemic when introduced control measures into the system. There are two ways to block the transmission of the epidemic: quarantine infected people to prevent further infection and closely monitor people who have contact with infected people and quarantine them once infected. Research shows these control measures effectively inhibit the spread of the disease.

G. Chowell *et al.* built SEI*JR model* [4] based on differential equations of epidemic, where *J* stands for confirmed case. The simulation results corresponded with numerical data.

B. Model Based on Network Dynamics

Meyers and Newman *et al.* published "*Network theory and SARS: Predicting outbreak diversity"* [5]. On the basis of data analysis of different cities, they believed that many infectious diseases spread through populations via the networks formed by physical contacts among individuals. They found the threshold for the epidemic to develop from outburst to prevalence and explained why SARS only prevails in some certain cities.

Lin Guoji *et al.* proposed *the small world network model* to predict SARS infection in 2003 [6]. Other methods for epidemic research based on network dynamics include

cellular automaton, artificial neural network, and scale-free network.

C. Model Based on Statistic Information

Statistic models simulate the process and obtain equations by fitting the existing data. They are usually based on the current situation and a retrospective analysis of cases in the stricken area. Therefore, the relationship obtained only has local significance. The accuracy of prediction will be limited by empirical value of stricken areas.

Wang and Ma (2007) employed simple linear regression model to study the characteristics and trends of AIDS in Hong Kong. Feng and Bai (2005) used time series model to predict the trends of AIDS in Shenyang, China. It turned out that ARIMA model fitted well. ARIMA model is simple and accurate, which is suitable for mid and short-term prediction.

Markov model [7] is also used in epidemic prediction. However, *Markov model* is only applicable to short-term prediction as transition matrix will change over time.

III. Methodology

We build a K-stage dynamic decision-making model based on *SIQR model*. We comprehensively consider various factors in our model including spread of the disease, the quantity of medicine needed, delivery systems, manufacturing speed of vaccine and drug and other factors such as the cost of delivery.

People are divided into four groups: **susceptible people, infected people, quarantined people and people removed from the system.** In our model, $S(t)$ stands for the total number of susceptible people at time t; $I(t)$ stands for total number of infected people at time t; $Q(t)$ stands for total number of quarantined people at time t and $R(t)$ is the number of people removed from the system at time t. R consists of two groups of people: people recovered from the epidemic and people who have been vaccinated.

There are several assumptions in our model:

1) The manufacturing company is able to produce both drugs and vaccines.
2) T is the length of each time period, which is fixed in our model. Each infected patient needs *one unit* of drugs, and after one time period, he has the possibility of γ_2 to be recovered from the epidemic
3) Each susceptible person needs *one* vaccine, and he has the possibility of μ to be immune to Ebola.
4) We do not introduce asymptomatic period into our model and treat people who are in asymptomatic period as susceptible people. Patients remain asymptomatic for a period of 2-21 days, and during this time tests for the virus will be negative, and patients are not infectious, posing no public health risk. Also, vaccination on people in asymptomatic period will work as well [8].

Symbols and notations are shown in the Table I below:

TABLE I: Symbols and Notations

Symbol	Definition	Units
$S(t)$	Total number of susceptible people at time t	number
$I(t)$	Total number of infected people at time t	number
$Q(t)$	Number of quarantined people at time t	number
$R(t)$	Number of people removed from the system at time t	number
T	The length of each time period	Days
s_{xki}	The actual supply of drugs to area i at the beginning of time period k	Per unit
s_{yki}	The actual supply of vaccine to area i at the beginning of time period k	Per unit
I_{ki}	the consumption of drugs in area i during time period k	Per unit
w_{ki}	the consumption of vaccine in area i during time period k	Per unit
μ	possibility of being immune to Ebola after being vaccinated	Unitless
N	The total population of the affected areas	Number
λ	Possibility that an infected person will infect a susceptible person during the time period t	Unitless
$p_0(t)$	the percentage of quarantined people to the total number of infected people	Unitless
β	Natural birth rate of the population	Unitless
ξ	Natural death rate of the population	Unitless
θ	the recovery rate of the quarantined people	Unitless
α_1	the rate of death due to Ebola of infected people	Unitless
α_2	the rate of death due to Ebola of quarantined people	Unitless
γ_1	the recovery rate of the infected without medication	Unitless
γ_2	the recovery rate of the infected using drugs	Unitless
p	Vaccination rate	Unitless
DX_{ki}	The demand of drugs at the end of time period k in area i.	Per unit
DY_{ki}	The demand of vaccine at the end of time period k in area i.	Per unit
M	Number of factories manufacturing drugs and vaccine	Number
N	Number of affected areas	Number
V_{Xj}	The maximal speed of producing drugs of factory j	Per unit
V_{Yj}	The maximal speed of producing vaccine of factory j	Per unit
D_{xkj}	Actual production of drugs of factory j	per unit
D_{ykj}	Actual production of vaccine of factory j	per unit
c_{ij}	cost of delivering drug and vaccine from factory j to area i.	per unit
X_{ij}	shipments of drugs from factory j to area i	per unit
Y_{ij}	shipments of vaccine from factory j to area i	per unit

First of all, **we divide the process into different time periods**. As is shown in the diagram below, **each time period starts from the solid line and ends at the dash line**. The length of each time period is T.

Without loss of generality, we take the situation in area i during time period k as an example to elaborate our model. The situation in other area during different time period can be studied in the same manner.

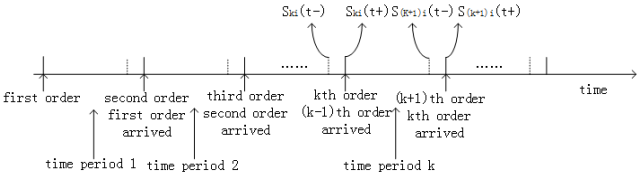

Fig. 1. Order schedule.

At the beginning of time period k, s_{xki} drugs and s_{yki} vaccines will arrive in area i. Given that there are no drugs and vaccines arriving at the beginning of time period 1, $s_{xk1} = 0, s_{yk1} = 0$.

During time period k, the consumption of drugs is l_{ki} and the consumption of vaccine is w_{ki} in area i. We assume that drugs and vaccine arriving at the beginning of time period k will be used up during the period. Therefore,

$l_{ki} = s_{xki}, w_{ki} = s_{yki}$.

Each infected patient needs *one unit* of drugs and each susceptible person needs *one unit* of vaccine.

The K-stage dynamic decision-making model includes four processes:

1) Update the initial value of time period *k*;
2) Predict the demand of drugs and vaccine at the end of time period *k* by using *SIQR model*;
3) Obtain the actual supply of drugs and vaccine after considering the maximal manufacturing speed of factories;
4) In order to minimize the cost, by employing Linear Programming Method, we can acquire the quantity of drugs and vaccine delivered from each factory to each area.

We will elaborate on the four processes in the following subsection.

A. Update the Initial Value of Time Period k

$$\begin{cases} S_{ki}(t^+) = S_{ki}(t^-) - \mu w_{ki}, \\ R_{ki}(t^+) = R_{ki}(t^-) + \mu w_{ki}, \\ I_{ki}(t^+) = I_{ki}(t^-), \\ Q_{ki}(t^+) = Q_{ki}(t^-), \\ w_{ki} = s_{yki}. \end{cases} \tag{1}$$

μ is the possibility of being immune to Ebola after being vaccinated. In formula (1),

$$\begin{aligned} S_{ki}(t^+) &= \lim_{\lambda\to 0^+} S_{ki}(t+\lambda), S_{ki}(t^-) = \lim_{\lambda\to 0^-} S_{ki}(t+\lambda), \\ R_{ki}(t^+) &= \lim_{\lambda\to 0^+} R_{ki}(t+\lambda), R_{ki}(t^-) = \lim_{\lambda\to 0^-} R_{ki}(t+\lambda), \\ I_{ki}(t^+) &= \lim_{\lambda\to 0^+} I_{ki}(t+\lambda), I_{ki}(t^-) = \lim_{\lambda\to 0^-} I_{ki}(t+\lambda), \\ Q_{ki}(t^+) &= \lim_{\lambda\to 0^+} Q_{ki}(t+\lambda), Q_{ki}(t^-) = \lim_{\lambda\to 0^-} Q_{ki}(t+\lambda). \end{aligned} \tag{2}$$

We assume that the process of vaccination completes instantly, and after an incredibly short time, the number of susceptible people(S) and people removed from the system(R) will change accordingly [9].

B. Predict the Demand of Drugs and Vaccines at the End of Time Period k

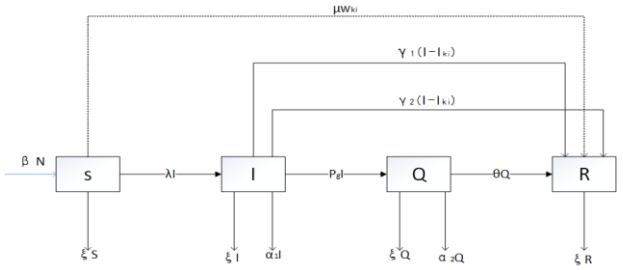

Fig. 2. *SIQR Model* considering drugs and pulse vaccinating.

Comprehensively considering the rate of birth and death, different recovery rates of patients receiving different medical treatment, the infection rate, etc, we build the *SIQR model* to predict the spread of Ebola:

(The meaning of each parameter can be found in Table I)

$$\begin{cases} \dfrac{dS}{dt} = \beta N - (\lambda I(t) + \xi) S(t), \\ \dfrac{dI}{dt} = \lambda S(t) I(t) - (p_q + \xi + \alpha_1) I(t) - \gamma_1 (I - l_{ki}) - \gamma_2 l_{ki}, \\ \dfrac{dQ}{dt} = p_q I(t) - (\xi + \alpha_2 + \theta) Q(t), \\ \dfrac{dR}{dt} = \theta Q(t) - \xi R(t) + \gamma_1 (I - l_{ki}) + \gamma_2 l_{ki} \\ l_{ki} = s_{xki}. \end{cases} \tag{3}$$

By solving the differential equation set above, we can obtain the number of susceptible people, infected people, quarantined people and people removed from the system at the end of time period *k*, which is also the number of each group of people just before time period *k*+1. They can be represented as $S_{(k+1)i}(t^-)$, $I_{(k+1)i}(t^-)$, $Q_{(k+1)i}(t^-)$, $R_{(k+1)i}(t^-)$ respectively.

Then we can obtain the demand of drugs DX_{ki} and vaccine DY_{ki} at the end of time period *k* in area *i*:

$$DX_{ki} = I_{(k+1)i}(t^-), \quad DY_{ki} = pS_{(k+1)i}(t^-). \tag{4}$$

In formula (4), *p* is the vaccination rate of susceptible people.

C. Obtain the Actual Supply of Drugs and Vaccine at the Beginning of Time Period k+1

Assuming there are *M* manufacturing factories and *N* affected areas, V_{Xj} is the maximal manufacturing speed of drugs of factory *j* and V_{Yj} is the maximal manufacturing speed of vaccine of factory *j*.

D_{xkj} is the actual production of drugs of factory *j* during time period *k*, and D_{ykj} is the actual production of vaccine of factory *j*.

$s_{x(k+1)i}$ is the actual supply of drugs to area *i* at the beginning of time period *k*+1, and $s_{y(k+1)i}$ is the actual supply of vaccine to area *i*.

1) As to drugs

$$D_{xkj} = \begin{cases} V_{Xj} \mathrm{T}, & \sum_{j=1}^{M} V_{Xj} T \le \sum_{i=1}^{N} DX_{ki}; \\ \dfrac{V_{Xj}}{\sum_{j=1}^{M} V_{Xj}} \sum_{i=1}^{N} DX_{ki}, & \sum_{j=1}^{M} V_{Xj} T > \sum_{i=1}^{N} DX_{ki}; \end{cases}, j = 1, 2, \cdots, M \tag{5}$$

The function of D_{xkj} is piecewise. When all the factories manufacture drugs at their maximal speed during the time period, the production of drugs still cannot meet the total demand of affected areas. Under such situation, all the factories will actually produce drugs at their maximal speed. If the production of drugs can satisfy the demand, factories will not have to produce drugs at their maximal speed and they just need to produce the amount of drugs needed at the end of the time period.

We assume the drugs will arrive at the beginning of time period *k*+1, then the function of $s_{x(k+1)i}$ can be obtained:

$$S_{x(k+1)i}=\begin{cases}\dfrac{DX_{ki}}{\sum_{i=1}^{N}DX_{ki}}\sum_{j=1}^{M}V_{Xj}T, & \sum_{j=1}^{M}V_{Xj}T\le\sum_{i=1}^{N}DX_{ki};\\ & ,i=1,2,\cdots,N\\ DX_{ki}, & \sum_{j=1}^{M}V_{Xj}T>\sum_{i=1}^{N}DX_{ki};\end{cases}\tag{6}$$

The actual supply to area *i* is a piecewise function as well. When the total production is able to meet the total demand, area *i* will receive the drugs it need. Otherwise, drugs will be distributed to different affected areas according to the proportion of one's needs.

2) As to vaccine

The actual production and distribution of vaccine are analogous to the situation of drugs. We can also obtain the functions of D_{ykj} and $S_{y(k+1)i}$:

$$D_{ykj}=\begin{cases}V_{Yj}\mathrm{T}, & \sum_{j=1}^{M}V_{Yj}T\le\sum_{i=1}^{N}DY_{ki};\\ \dfrac{V_{Yj}}{\sum_{j=1}^{M}V_{Yj}}\sum_{i=1}^{N}DY_{ki}, & \sum_{j=1}^{M}V_{Yj}T>\sum_{i=1}^{N}DY_{ki};\end{cases},\ j=1,2,\cdots,M\tag{7}$$

$$S_{y(k+1)i}=\begin{cases}\dfrac{DY_{ki}}{\sum_{i=1}^{N}DY_{ki}}\sum_{j=1}^{M}V_{Yj}T, & \sum_{j=1}^{M}V_{Yj}T\le\sum_{i=1}^{N}DY_{ki};\\ & ,i=1,2,\cdots,N\\ DY_{ki}, & \sum_{j=1}^{M}V_{Yj}T>\sum_{i=1}^{N}DY_{ki};\end{cases}\tag{8}$$

D. Acquire the Quantity of Drugs and Vaccine Delivered from Each Factory to Each Area

In order to minimize the cost when delivering drugs and vaccine from manufacturing factory to affected area, we employ Linear Programming Method to determine the quantity.

X_{ij} denotes the shipments of drugs from manufacturing factory *j* to area *i*. Y_{ij} is the shipments of vaccine from factory *j* to area *i*.

c_{ij} is the cost of delivering drugs and vaccine from manufacturing factory *j* to area *i*.

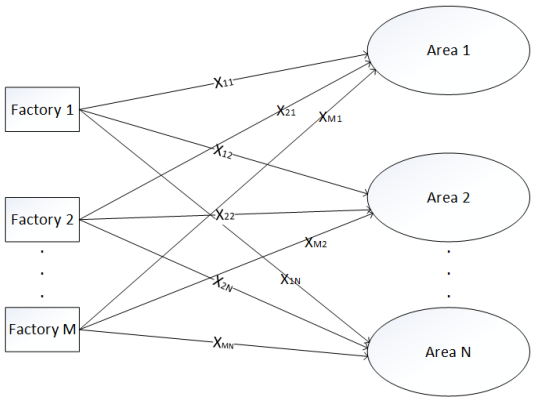

Fig. 3. Delivery system.

1) As to drugs

$$objective:\min\sum_{i=1}^{N}\sum_{j=1}^{M}c_{ij}X_{ij}$$

$$s.t.\begin{cases}D_{xkj}=\sum_{i=1}^{N}X_{ij}, & j=1,2,\cdots,M,\\ S_{x(k+1)i}=\sum_{j=1}^{M}X_{ij}, & i=1,2,\cdots,N,\\ X_{ij}>0, & 0<i\le N,0<j\le M,i,j\in Z\end{cases}\tag{9}$$

By solving Linear Programming Problem (9), we can know the quantity of drugs delivered from factory *j* to area *i*, $i=1,2,\cdots,N;\ j=1,2,\cdots,M$.

2) As to vaccine

The situation of vaccine is analogous to drugs.

$$objective:\min\sum_{i=1}^{N}\sum_{j=1}^{M}c_{ij}Y_{ij}$$

$$s.t.\begin{cases}D_{ykj}=\sum_{i=1}^{N}Y_{ij}, & j=1,2,\cdots,M,\\ S_{y(k+1)i}=\sum_{j=1}^{M}Y_{ij}, & i=1,2,\cdots,N,\\ Y_{ij}>0, & 0<i\le N,0<j\le M,i,j\in Z\end{cases}\tag{10}$$

By solving Linear Programming Problem (10), we can obtain the quantity of vaccine delivered from factory *j* to area *i*, $i=1,2,\cdots,N;\ j=1,2,\cdots,M$.

IV. Data Simulation

First of all, we have to quantify the term "eradicating Ebola". When the number of susceptible people, infected people and quarantined people remains stable and is smaller than the threshold for some periods, we assume that the epidemic has been effectively controlled, which is consistent with common sense.

Some inputs of our model are hard to obtain, such as the initial number of susceptible people and quarantined people of Ebola in affected areas like Guinea, Liberia and Sierra Leone. Also, there is no official information about drugs and vaccine used to resist Ebola. Therefore, we assume some of the value by experience.

We will explain the data we set up. Then the operating results will be given. Finally we will analyze the results according to the actual situation.

A. The Data Set

Considering there is no enough manufacturing company which has the ability to produce drugs of Ebola, we assume that there are only 2 manufacturing companies. The three worst-affected areas are Guinea, Liberia and Sierra Leone. Thus the number of affected areas is 3.

$$M=2,\ N=3.$$

When programming, we divide each time period into two parts in order to highlight the sudden change at the beginning of each time period. The first order of drugs and vaccine will arrive at time t_0. The initial number of susceptible people, infected people, quarantined people and people removed from the system of the three affected area are shown in Table

II. The parameters are also presented in Table II.

We assume that the medical condition in Area 1 is the best, then Area 2. Area 3 is the worst.

TABLE II: INITIAL VALUE OF THE MODEL

	Area 1	Area 2	Area 3
$S(t_0)$	25409	10708	18090
$I(t_0)$	4578	1067	1822
$Q(t_0)$	3867	897	1458
$R(t_0)$	2009	468	799
p	0.15	0.15	0.15
μ	0.5	0.5	0.5
β	0.007	0.005	0.005
ξ	0.007	0.005	0.005
λ	0.0002	0.0003	0.0005
$p_q(t)$	0.8	0.7	0.9
γ_1	0.01	0.01	0.001
γ_2	0.9	0.9	0.9
α_1	0.6	0.6	0.6
α_2	0.5	0.6	0.4
N	1000000	1000000	1000000
θ	0.7	0.6	0.8
C_{i1}	1	2	2
C_{i2}	2	3	4

The maximal manufacturing speed of each factory is shown in Table III.

TABLE III: MANUFACTURING SPEED OF EACH FACTORY

	Factory 1	Factory 2
V_X	2000	3000
V_Y	600	700

Considering the different medical conditions of different areas, there are some slight differences in parameters among the three affected areas. Then we set up the delivery cost and speed of manufacturing drugs and vaccine of each factory by experience.

B. The Results of the Model

As for our data set, the epidemic in these three areas has been effectively controlled after 15 time periods.

Table IV shows the actual production of vaccine of the two factories and the actual amount of vaccine each area used in each time period.

TABLE IV: ACTUAL AMOUNT OF VACCINE PRODUCED AND USED

Time period	Factory 1	Factory 2	Area 1	Area 2	Area 3
1	0	0	0	0	0
2	1200	1400	393.62	338.44	1867.94
3	1200	1400	964.90	1075.05	560.05
4	1200	1400	867.86	1091.84	640.30
5	1200	1400	795.05	689.72	1115.23
6	1200	1400	882.44	869.73	847.83
7	1200	1400	882.06	947.63	770.31
8	1200	1400	846.23	818.45	935.32
9	1200	1400	868.17	856.72	875.11
10	1200	1400	874.99	895.44	829.57
11	1200	1400	860.51	854.48	885.01
12	1200	1400	866.01	860.16	873.83
13	1200	1400	869.93	876.17	853.90
14	1200	1400	865.01	864.25	870.73
15	1200	1400	866.05	863.55	870.40

Table V shows the quantity of vaccine delivered from each factory to each affected area and the delivery cost of the system in each time period.

TABLE V: THE QUANTITY OF VACCINE DELIVERED FROM EACH FACTORY TO EACH AREA AND THE DELIVERY COST

Time period	Y_{11}	Y_{12}	Y_{13}	Y_{21}	Y_{22}	Y_{23}	cost
1	0	0	0	0	0	0	0
2	935.12	893.45	2171.4	3281.8	2718.2	0	21783.0
3	0	0	3095.6	1369.8	769.96	2503.6	21255.0
4	625.36	716.04	2658.6	3103.0	2897.0	0	22272.0
5	654.4	884.13	2461.5	2837.4	3162.6	0	22508.0
6	0	0	3775.7	2803.6	2482.5	377.47	22116.0
7	334.51	426.67	3238.8	3087.6	2912.4	0	22578.0
8	453.98	595.31	2950.7	2955.9	3044.1	0	22590.0
9	157.72	237.76	3604.5	3084.9	2915.1	0	22757.0
10	269.07	359.81	3371.1	3073.3	2926.7	0	22658.0
11	351.82	462.26	3185.9	3015.8	2984.2	0	22632.0
12	254.12	346.5	3399.4	3055.0	2945.0	0	22691.0
13	269.89	362.93	3367.2	3060.0	2940.0	0	22670.0
14	308.5	409.29	3282.2	3038.1	2961.9	0	22653.0
15	278.83	374.7	3346.5	3048.5	2951.5	0	22673.0

Table VI shows the actual production of drugs of the two factories and the actual amount of drugs each area used in each time period.

TABLE VI: ACTUAL AMOUNT OF DRUGS PRODUCED AND USED

Time period	Factory 1	Factory 2	Area 1	Area 2	Area 3
1	0	0	0	0	0
2	4000	6000	4216.88	3611.70	2171.42
3	3095.61	4643.42	1369.82	769.97	5599.24
4	4000	6000	3728.39	3613.01	2658.60
5	4000	6000	3491.81	4046.72	2461.47
6	3775.71	5663.56	2803.58	2482.51	4153.18
7	4000	6000	3422.12	3339.06	3238.82
8	4000	6000	3409.91	3639.38	2950.71
9	4000	6000	3242.59	3152.89	3604.52
10	4000	6000	3342.38	3286.50	3371.12
11	4000	6000	3367.67	3446.41	3185.92
12	4000	6000	3309.10	3291.51	3399.39
13	4000	6000	3329.87	3302.96	3367.17
14	4000	6000	3346.64	3371.15	3282.21
15	4000	6000	3327.30	3326.22	3346.48

Table VII shows the quantity of drugs delivered from each factory to each affected area and the delivery cost of the system in each time period.

TABLE VII: THE QUANTITY OF DRUGS DELIVERED FROM EACH FACTORY TO EACH AREA AND THE DELIVERY COST

Time Period	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	Cost
1	0	0	0	0	0	0	0
2	935.12	893.45	2171.4	3281.8	2718.2	0	21783.0
3	0	0	3095.6	1369.8	769.96	2503.6	21255.0
4	625.36	716.04	2658.6	3103.0	2897.0	0	22272.0
5	654.4	884.13	2461.5	2837.4	3162.6	0	22508.0
6	0	0	3775.7	2803.6	2482.5	377.47	22116.0
7	334.51	426.67	3238.8	3087.6	2912.4	0	22578.0
8	453.98	595.31	2950.7	2955.9	3044.1	0	22590.0
9	157.72	237.76	3604.5	3084.9	2915.1	0	22757.4
10	269.07	359.81	3371.1	3073.3	2926.7	0	22658.0
11	351.82	462.26	3185.9	3015.8	2984.2	0	22632.0
12	254.12	346.5	3399.4	3055.0	2945.0	0	22691.0
13	269.89	362.93	3367.2	3060.0	2940.0	0	22670.0
14	308.5	409.29	3282.2	3038.1	2961.9	0	22653.0
15	278.83	374.7	3346.5	3048.5	2951.5	0	22673.0

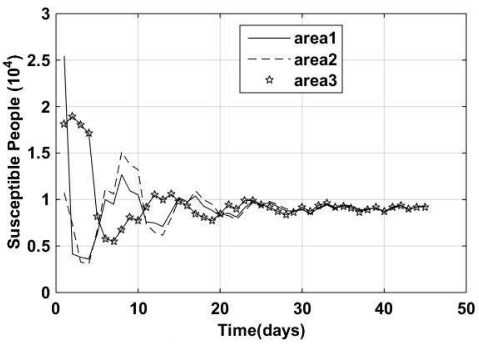

Fig. 4. The number of susceptible people in each time period.

The change in the number of the four groups of people in each time period is shown in Fig. 4-Fig. 7.

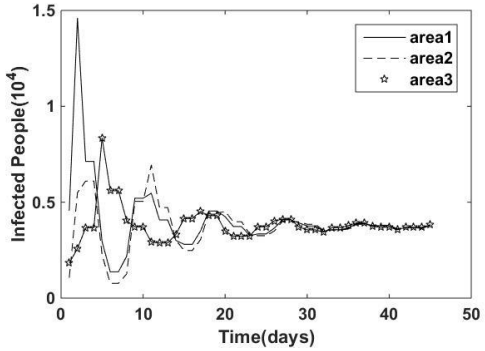

Fig. 5. The number of infected people in each time period.

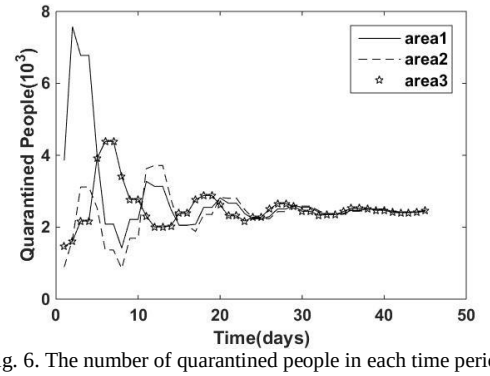

Fig. 6. The number of quarantined people in each time period.

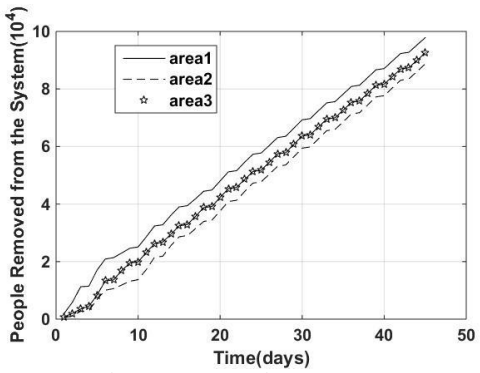

Fig. 7. The number of people removed from the system in each time period.

C. Analysis of the Results

As is shown in Fig. 4, the number of susceptible people reduces drastically in the first time period. Because there is no vaccine available in the first time period, many susceptible people become infected, while there are few newborns. Also, many infected people are been quarantined. Therefore, the number of infected people and quarantined people increases rapidly in the first time period. The epidemic gets worse.

In the following time periods, from period 2 to period 9, drugs and vaccine have arrived and are used by people in the affected areas. Many susceptible people are vaccinated. However, the epidemic situation is very serious at that time, and the vaccination rate p is not that high. The epidemic is just under preliminary control, but fluctuates largely.

From time period 9 to time period 15, with the wide use of drugs and vaccination, less susceptible people become infected and quarantined, and the epidemic is under control. The number of each group of people becomes stable, and fluctuates in an acceptable range.

After 15 time periods, the number of susceptible people, infected people and quarantined people remains stable and is smaller than threshold. Thus, the epidemic has been effectively controlled.

In conclusion, the results of the model are reasonable and are consistent with common sense.

V. Conclusions

The model in this paper has comprehensively considered many influencing factors, including the limit of manufacturing speed of drugs and vaccine and the different mechanisms of them. The process of vaccination completes instantly, while the treatment of drugs will last for a period of time.

We introduce different time periods into our model, and the number of susceptible people, infected people, quarantined people and people removed from the system in each time period can be derived from the differential functions in the model. The use of drugs and vaccine in each time period will influence the number of different groups of people in the next time period.

We have simulated the real situation of a medical system, and successfully obtained the actual change of the epidemic situation in each time period, the quantity of drugs and vaccine each manufacturing factory needs to produce and the optimal distribution plan of them.

Considering the limit of manufacturing speed and local medical condition, the production of drugs and vaccine cannot completely satisfy the demand of affected areas. The number of susceptible people, infected people and quarantined people will fluctuate a lot at first. With the use of vaccine and drugs in each time period, the change range of number of the three groups of people becomes smaller and tends to stabilize.

Because the quantity of drugs and vaccine used in three affected areas is equal, and the population and natural birth rate make little difference, the number of susceptible people, infected people and quarantined people at last is approximately the same. The number of people removed from the system shows a steady increase. The optimal distribution plan of each time period guarantees the best medical treatment effect and saves the cost.

The results turn out that the model in this paper can well fit the spread and the control of an epidemic, and is consistent with common sense and medical features.

For the convenience of the reader to reproduce the experimental results shown in this paper, we make our implementation of the improved *SIQR* model approaches involved in our evaluation available for download on the website:https://github.com/3Swordsman/The-code-of-our-paper-for-the-ICEMT-2015.git.

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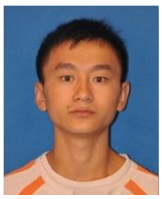

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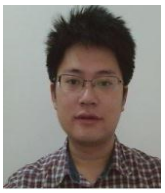

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