Standard Deviation to Standardize Values

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***Abstract*—The transformation of a quantitative variable to z-scores can also be called standard-deviation values. It measures the distance of a value from the mean in standard deviations. One purpose of the z-score is to help compare data values that are measured in different scales. With a given set of data values, the same z-scores will be computed regardless of the units of the values.**

***Index Terms*—Standard deviation, standardize values**

I. Defining Z-Scores

Highlight The transformation of a quantitative variable to z-scores can also be called standard-deviation values. It measures the distance of a value from the mean in standard deviations. One purpose of the z-score is to help compare data values that are measured in different scales. With a given set of data values, the same z-scores will be computed regardless of the units of the values.

Suppose a sample of n data values are given by

$$\{xi,\ i = 1, 2, \ldots, n\}$$

For these data we can compute the sample mean and sample standard deviation:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

The z-score is calculated by subtracting the sample mean from an individual raw score and then dividing the difference by the sample standard deviation:

$$z_i = \frac{x_i - \bar{x}}{s_x}$$

A z-score gives an indication of how unusual a value is because it tells how far it is from the mean of the data in units of the variable's standard deviation.

What we can also do with z-scores is that we can compare two z-score values from different data sets to judge which is more extreme. This comparison can be illustrated with z-scores corresponding to the performance scores of two athletes in different events.

Another example is to compute z-scores for the outcome of an examination, such as the SAT or ACT examinations. If the exams are comparable in measuring students' academic performance level, they should have about the same z-score for a given student.

Why do we need SAT or ACT? We have no objection to the argument that SAT or ACT was once one of the important admission indicators for US applications, and standardized tests are not perfect indicators. We've heard a lot of critiques about it - and how they perpetuate economic and racial discrimination. Having said that, in practice, without objective alternatives, we believe that the SAT or ACT are still necessary for the highly selective college admissions process, because without such a hard test, admissions officers can only rely solely on harder comparisons Variables. To name just one example, a 4.0 GPA at Thomas Jefferson High School for Science and Technology in Virginia is nothing like a 4.0 at Mira Coats High School, an excellent public school in California. It's much harder to get a 4.0 on Thomas Jefferson's most rigorous course [1].

Z-standardization is the procedure to transform absolute values, or ratings (e.g., 1 = *don't agree at all* to 7 = *totally agree*) to relative scores that reflect each answer's rank in comparison to the ranks of all responses in that sample. In z-standardization, the sample mean score is subtracted from each single observation, and this difference is then divided by the sample's standard deviation. The result is a scale where a score of 0 means that this observation was at the sample's mean level, and a z-score of 1 reflects an observation one standard deviation above the sample mean [2].

While some have been outspoken about how colleges need to eliminate standardized testing in the admissions process, there are those who do argue that the tests? though definitely flawed? Do provide a valuable reference in the decision-making process. In fact, a recent article titled "Why All Kids Should Take the SAT: Students' Defense of Standardized Tests" states, "To be sure, the SAT and ACT aren't perfect? But compared to those more subjective measures (like essays, school interviews, and letters of recommendation), it at least provides a quantitative measurement way, it has consistently been shown that standardized tests are good predictors of success on campus and throughout life, even taking into account the socioeconomic background of test takers. And because they are also strongly correlated with IQ, test scores are the type of indicator that admissions officers should be able to refer to, from the admissions process Eliminating that objectivity in China would be disastrous."

For linear and generalized linear models, coefficients are the currency of the realm in scientific interpretations. Statistical modeling involves specification, estimation, and model selection, all designed to obtain coefficient estimates that can serve as trustworthy measures of the effects of predictors on responses [3].

In stark contrast to MIT is Cal tech, which has extended the waiver requirement for SAT and ACT test scores already by two years. The College will continue to study the impact of standardized grades on the long-term success of students.

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This year, Cal tech's policy is Test-Blind right now, and even if you submit standardized test scores, Cal tech will not consider those scores in the admissions process.

In fact, that's why MIT announced in March that they would be reinstating the SAT or ACT testing requirements in their admissions process after the pandemic period - for the upcoming admissions cycle, MIT requires applicants to submit the SAT/ACT scores. As Stu Sch mill, MIT's longtime head of admissions, said at the time: "After careful consideration, we have decided to reinstate our SAT/ACT requirements in future admissions cycles. Our research shows that standardized tests can help us better Assessing the academic readiness of all applicants also helps us identify socioeconomically disadvantaged students who would otherwise demonstrate their readiness for MIT who do not have access to advanced coursework or other enrichment opportunities. In our view, requirements are more important than optional Testing policies are fairer and more transparent? But months have passed, and America’s elite universities have not followed MIT’s lead. Why? Because America's elite universities have historically tended to emulate Harvard, not MIT. Harvard has announced that it will keep standardized testing optional through at least the class of 2030.

II. Applications Using Z-Scores

Z-score is often used in real life for different purposes. Just like mentioned earlier, it's a useful tool to measure score of the same student in two different tests, for example SAT and ACT. Or just a regular test from the student's class. If this student got 80 out of 100 for test A and only 70 in test B. It seems like the student has not performed well enough in test B than test A. However, that’s not always true because of other factors might affect the test score such as the difficultly of the test, the time given used to complete the test. So, it's unfair to simply compare the scores and say test B was done poorly. In this case, using z-score will help us understand which test the student has done better on [4].

Now assume we know the data we need to know for using z-score, and that will be the mean and stander deviation for both tests:

$$\mu A=90,\ \sigma A=10;\ \mu B=60,\ \sigma B=5$$

And now we can use the equation for z-score:

$$z_i = \frac{x_i - \bar{x}}{s_x}$$

The z-score for test A will be:

$$z_A = \frac{80-90}{10} = -1$$

The z-score for test B will be:

$$z_B = \frac{70-60}{5} = 2$$

And we can see that even though the student got a lower score in test B than test A, however the z-score is higher, that means the student not only didn't perform poorly in test B, but also done better than test A.

Manage test scores is a great use of z-score, but it's not the only application. Z-scores can also be used in statistic models-normal models for example. This is a typical example of a normal model (see below), produced by a French mathematician, Abraham de Moivre in 1733. This model for unimodal, symmetric data gives us information because it tells us how likely it is to have z-scores between inverse numbers like -1 to 1.

```
> n=1000 #sample size
> x=rnorm(n, mean=0, sd=1) # mean and sd of a standard normal distribution
> length(x)

[1] 1000

> head(x)

[1] -1.5645890 -1.0556493 -0.8407620 1.9918189
1.2788311 -0.5225864
```

To study the entire population is time and resource intensive and not always feasible; therefore studies are often done on the sample; and data is summarized using descriptive statistics. These findings are further generalized to the larger, unobserved population using inferential statistics [5].

Sometimes z-score can also be used in different athletic subjects, for example soccer and basketball, it's unfair to compare the points scored by the team, because basketball games usually end with a much larger score than soccer. On Jan 18, 2021 the NBA team Worrier had a game match with Lakers and the score of that game was 115:113. 6 days later in UEFL, team Manchester City won their game against Paris Saint-Germain with a 2:1. Does that means the soccer players are over all worse athletes than basketball players? Not necessarily, but using z-score can help us understand overall who is scoring more points, soccer players or basketball players. From 2020-2021, the average score in a professional soccer game is 2.69 and the stander deviation is around 2.13. We can plug these numbers in to the z-score equation and we will get [6]?

$$\mu=2.69,\ \sigma=2.13$$

And using the total score 3 in the Man City vs. PSG game, we get:

$$z = \frac{2-2.69}{2.13} = -0.324$$

Now, the average score of professional basketball teams is around 109.8, the stander deviation will be about 8.3, using these data we will be getting:

$$\mu=109.8,\ \sigma=8.3$$
$$z = \frac{115-109}{8.3} = 0.723$$

So, in this example, the basketball team has a higher z value for their game than the soccer team with a difference around 1. However, the numbers can vary, if a basketball team scored less than the mean of average score or the soccer team had score higher than their mean score, the z value can be reversed. Using the z-value we can actually compare and see the differences between different events. Sometimes it can even be used on the same sport. if you want to compare two players on the same team but playing different positions, z-score can also be used, like goalies and attackers in soccer. We can collect their data on their specific field, convert them in to z-score and compare to see who did better in the season.

This graph has 1000 random samples within it, and we can plot the series of values in the sample against their index.

We can also plot the histogram using a density scale (so area of histogram equals). This sample of values has a histogram as follows: Fig. 1.

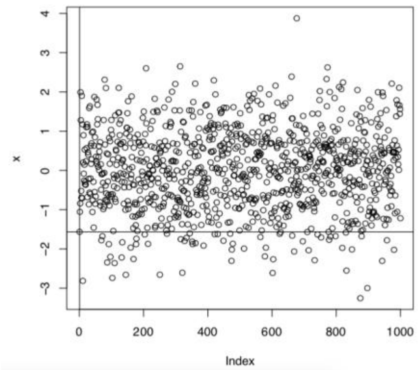

Fig. 1. Plot the histogram using a density scale.

This sample of values has a histogram as following graphs: Fig. 2 and Fig. 3.

```
[1] -1.564589
```

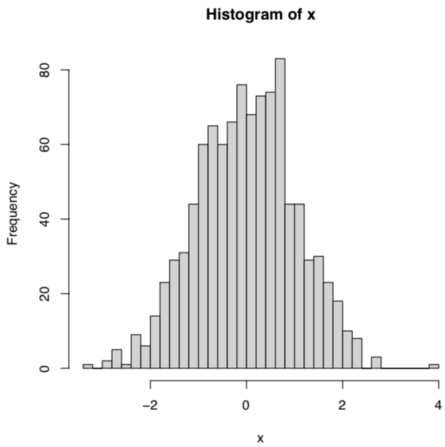

Fig. 2. Sample of values in a histogram

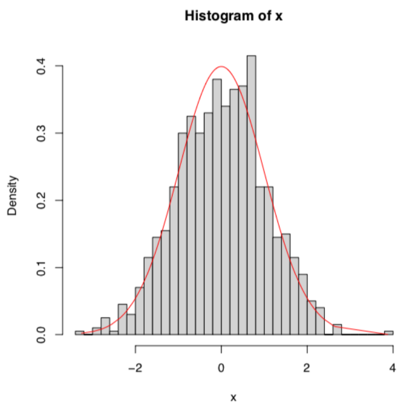

Fig. 3. Red line shows the normal model reaching toward multiple stander deviations.

On the last graph we can see a red line, and that is the normal model reaching toward multiple stander deviations, nearly symmetry, which proven a rule came up by Abraham de Moivre-68-95-99.7 Rule, describing the percentage within numbers of deviations.

There is other use of z-score still, like deciding which ranking list made by different organizations to look at. Each school has their own strength and weaknesses and different organizations looks at them differently. For example, US News; According to the latest news from BENJAMIN NICHOLSON, spokesman for US News Global Education, the latest list of US News' 38th comprehensive ranking of American universities and the list series of undergraduate professional rankings of American universities will be officially released on September 12. US News has been working on college rankings since 1983, with the main goal of helping students find the best college for them and compare the important factors behind different colleges. This year's university rankings are measured by factors such as the university's teaching achievement, faculty, expert opinion, financial resources, alumni giving and the performance of incoming students. In order to provide you with more abundant resources, this year, in addition to the comprehensive rankings, ranking lists of public schools, liberal arts colleges and three majors will also be released, including ranking lists of engineering, business and computer science.

US News is the most valuable and heavyweight list in the rankings of American colleges and universities. In the previous edition of the rankings, US News refused to remove the SAT and ACT scores from the rankings, but made minor adjustments to their weights., Although the final ranking of the top universities has little influence, many later schools still have a relatively large influence, such as: University of Michigan, Ann Arbor, New York University, Georgia Tech, etc. Let's wait and see how the rankings will change this year.

III. Z-Score in Future

If Standard deviation is like a ruler used in statisticians, they use it to find useful information within large quantity samples, and z-score, or in other words, standard deviation values, is the key of using stander deviation. Z-score can be used in nearly everything that can be managed or examined by statistic. But it's most helpful when we use it to measure and compare samples without quantitative variables. Therefore, it can be and certainly will be used in more fields that requires observation among different variables.

- using z-scores to compare athletic performances comment/discuss how you change the sign of a performances measure so that high values are better
- using z-scores to compare standardize test scores with different test
- discuss z-scores for normal models(section5.3), explain 68-95-99.7 rule
- using z-scores to compare an empirical distribution to the normal model

The normal probability plot displays the scatter plot of sample values vs. the theoretical normal z-scores. this plot is equivalent to the plot of the samples z-scores vs. theoretical normal z-scores. Fig.4.-Fig.6.

```
> x=rnorm(100,mean=65,sd=6)
> head(x)
```

[1] 64.28120 73.92985 71.70808 63.44836 60.44013 65.14021

```
> hist(x,breaks=50)
> # Display normal probability plots of x and of x s z scores
>
> par(mfcol=c(2,1))
> # Plot of ordered sample values versus the theoretical average value
> # of ordered N(0,1) sample
> qqnorm(x)
> # Add line with slope = sample st dev and intercept = sample mean
>
> abline(a=mean(x), b= sqrt(var(x)), col= red )
> #Compute z-scores of x
>
> zscores= (x- mean(x))/sqrt(var(x))
> qqnorm(zscores)
> abline(a=0, b=1, col= blue )
> # Add line with slope =1, intercept =0
> # Only difference in plots are the y axis units
> # First is units of x, second is units of z scores
> #
>
>
```

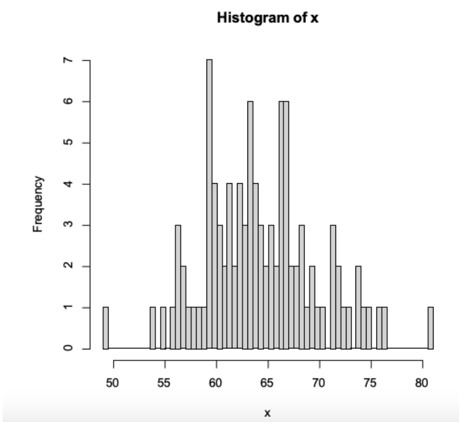

Fig. 4. Histogram of x.

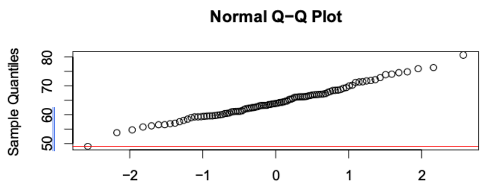

Fig. 5. Theoretical quantiles.

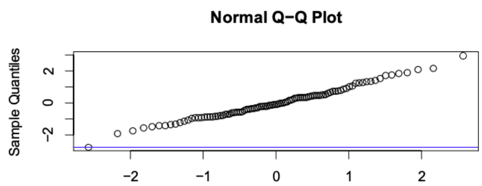

Fig. 6. Theoretical quantiles.

In a normal probability plot, which is created with the R function qqnorm(), a sample is consistent with coming from a normal model if the points lie close to a line. With the original sample, the line has slope equal to the standard deviation and intercept equal to the sample mean. With the z scores, the line has slope equal to 1 (standard deviation of z-scores) and the intercept equals 0 (mean of z-scores).

Note: if Z is a vector of sample values from a Normal (0,1) distribution, i.e., the standard normal distribution, then for constants mu and sigma, X= mu + sigma*Z is a sample from a Normal(mu,sigma) distribution.

Illustrate how normal probability plots vary from sample to sample with data that comes from a normal model. Fig.7.

```
> ntrials=6
> par(mfcol=c(3,2))
> # Use set.seed() to replicate samples generated
> set.seed(1)
> for (j in c(1:ntrials)){
+ x=rnorm(200,mean=65,sd=6)
+ qqnorm(x)
+ title(sub=paste("Trial ", j,sep=""))
+ # Add line with slope = sample st dev and intercept = sample mean
+
+ abline(a=mean(x), b= sqrt(var(x)), col= red )
+
+ }
> ntrials=6
> par(mfcol=c(3,2))
> set.seed(1)
> for (j in c(1:ntrials)){
+ x=rnorm(200,mean=65,sd=6)
+ hist(x, breaks=30)
+ title(sub=paste("Trial ", j,sep=""))
+ # Add line with slope = sample st dev and intercept = sample mean
+
+ abline(a=mean(x), b= sqrt(var(x)), col= red )
+
+ }
```

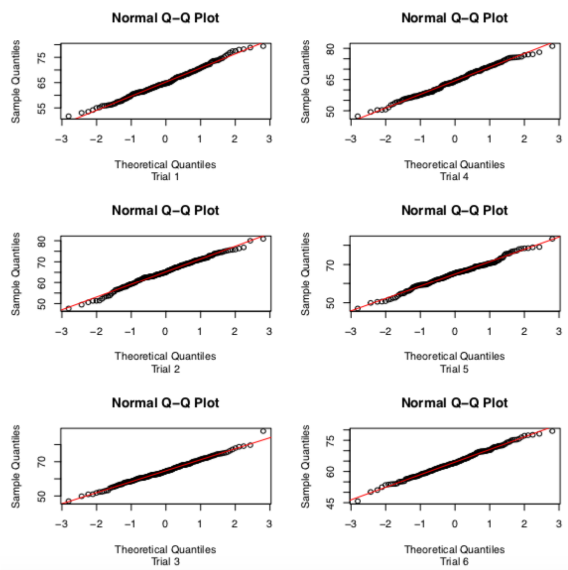

Fig. 7. Theoretical quantiles trial 1-6.

Another of the more common applications is in accounting and financial analysis, such as the Atman Z-score model. Edward Altman, a professor at the Stern School of Business at New York University, observed the bankrupt and non-bankrupt production enterprises in the United States in 1968, using 22 financial ratios and mathematical statistical screening to establish the famous 5-variable Z-score model Z-score model is based on a multivariate statistical method, taking the bankrupt enterprise as a sample, through a large number of experiments, the operation of the enterprise, the analysis of bankruptcy or not, the system of judgment. The Z-score model is widely used in worldwide, such as United States, Australia, Brazil, Canada, the United Kingdom, France, Germany, Ireland, Japan and the Netherlands [7].

Inferential tests on baseline variables in non-randomized studies can also be troublesome, albeit for a different reason. While it is theoretically justifiable to test for differences in population parameters within a non-randomized sample, the results from these tests are largely dependent on sample size and can be difficult to interpret (e.g., propensity-score matched studies [8].

Standard deviation is an especially useful tool in investing and trading strategies as it helps measure market and security volatility—and predict performance trends. As it relates to investing, for example, an index fund is likely to have a low standard deviation versus its benchmark index, as the fund's goal is to replicate the index [9].

The biggest advantage of Z-score is that it is simple and easy to calculate, and many tools, do not need to load packages, and can calculate Z-scores and compare them with only the simplest mathematical formulas. In addition, Z-score can be applied to numeric data and is not affected by the magnitude of the data, because its role is to eliminate the inconvenience of the magnitude to the analysis.

But Z-score applications are also risky, here is the limitations and disadvantages of the Z-Score model? First, estimating z-score requires the mean and variance of the population, but this value is difficult to obtain in real-world analysis and mining, and in most cases is replaced by the mean and standard deviation of the sample. only 2 extreme cases (default and no default) are considered, and no more detailed classification is made for debt restructuring or high recovery rates despite defaults. Second, the weight may not always be fixed, must be adjusted frequently. The last, Z-score has certain requirements for the distribution of data, and the normal distribution is the most conducive to Z-score calculation.

Methods based on summary statistics (minimum, maximum, lower quartile, upper quartile, median) reported in the literature facilitate more comprehensive inclusion of randomised controlled trials with missing mean or variability summary statistics within meta-analyses [10].

Conflict of Interest

The author declares no conflict of interest.

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