Optimal Design of Government Guarantee and Revenue Cap Agreements in Public — Private Partnership Contracts

Hongyu Jin and Chunlu Liu

***Abstract*—Public–private partnerships (PPP) have been widely used in delivering infrastructure projects as they mobilise social capital to participate in infrastructure construction. However, the long operational period of PPP projects draws high market risks, which deters investment in PPP projects. Government guarantees are frequently used as an investment incentive as they reduce the probabilities of suffering loss for social participants. Nevertheless, government guarantees cannot fully control the overly lucrative conditions for private investors, which is the reason that revenue cap agreements are designed as a supplement in PPP contracts. This research proposes a methodology that can be used to design the specific thresholds for triggering such combined agreements, i.e. government guarantee and revenue cap agreements. These government guarantee and revenue cap decision models adopt geometric Brownian motion modelling as a data analysis tool and the fair preferences in relation to the project profits held by the project parties as an indicator in finding the optimal value of combined agreements. In addition, based on the project parties' capabilities to bear risk, a self-regulation process for the value of combined agreements is created to ensure the levels of risk borne by the project parties are within their acceptable ranges. The research outcome shows that the proposed methodology in this paper is effective and able to determine the optimal value of government guarantee and revenue cap agreements.**

***Index Terms*—Contract design, government guarantee, public–private partnership, real option, revenue cap.**

I. Introduction

Over the past decades, public–private partnerships (PPP) have gained popularity in delivering public infrastructure. Daube *et al.* (2008) defined a PPP project as "a long-term contractual arrangement between the public and private sector to realize public infrastructure and services more cost effectively and efficiently than under conventional procurement". Due to their strength in delivering infrastructure projects, governments have been introducing social capital through PPP agreements to alleviate financial pressure [1]. Project investors (referred to as 'concessionaires' in this paper) expect to gain profits by operating projects. However, the implementation of PPP projects is not without challenges over a long operational period. Market surroundings continue changing throughout project life cycles and critical decisions made during the pre-construction stage tend to neglect the influence of market uncertainties [2], which may lead to ex-post renegotiation or even project failure. To cope with market uncertainties, concessionaires often request government support to share financial risks. Also, since a project with excessive market risk will find it difficult to attract social capital, government support plays the role of providing an incentive for concessionaires to participate in PPP projects [2]. Government guarantees are one of the most common forms of government support, and they define the minimum level of annual net income. If annual net income is lower than the pre-specified value of the government guarantee, the government will compensate for the deficit [3]. Nevertheless, large government guarantees can be a fiscal burden on the government and society [4], which is one reason that governments always try to ensure the granting guarantees are off the balance sheets according to local accounting standards.

Accounting standards for PPP projects remain controversial. According to the International Financial Reporting Interpretation Committee 12, project firms' balance sheets should include PPP assets and liabilities only if they control the assets [5]. The European PPP Expertise Centre (2010) claimed that if the value of a guarantee is more than 50% of the capital investment, then it should be counted in the balance sheet. The Australian Heads of Treasuries Accounting and Reporting Advisory Committee offered a different view, suggesting that PPP liabilities should be listed on the balance sheet of the party who bears the majority of risk [6]. Governments should decide which accounting standard to follow, normally the local one, to avoid suffering a fiscal burden.

Another way to protect public participants from fiscal problems is to propose a revenue cap agreement that defines the upper limit of the annual net income for concessionaires [7]. The profitability of the project for concessionaires should be improved with a government guarantee and some of the financial risk is transferred from the concessionaires' side to the public sector [8]. As governments then share more financial risk via government guarantee agreements, in return governments should have the right to share the excess profits [9]. Through this profit sharing machinism, overly lucrative conditions for concessionaires are expected to be regulated. In addition, when the net present value (NPV) of a project for concessionaires is higher than the NPV for governments, the income gap between these project parties can be controlled.

Government guarantees and revenue caps are often evaluated separately and few research studies have integrated revenue caps with government guarantees to study their interaction [10]. Even though some studies have considered government guarantees and revenue caps as combined agreements, these studies mostly calculated the option value

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of combined agreements without giving their specific levels [3], [11]. In order to overcome these above limitations, this study proposes a methodology to design the thresholds for triggering government guarantees and revenue caps for project parties. The aim in designing the government guarantee and revenue cap is to reach a balance between the project parties on project revenue and risk. However, when these two agreements work together, they counteract each other. It is expected that the higher the value of the government guarantee, the lower the value of the revenue cap as a response, to average the risk, which means that the values of the government guarantee and the revenue cap should be decided carefully with full consideration of the interaction between them. Thus, a self-regulation process is here established to adjust their values in order to control the project risk for both parties within an acceptable range.

II. Theoretical Background

A. NPV Analysis for PPP Projects

Project participants usually forecast the profitability of a potential project using NPV analysis. The first step of this approach is to clarify the cash inflows and outflows during the project life cycle. A typical project life cycle for a PPP project is shown in Fig. 1, where t_0 is the commencement date of the project, t_c indicates the instant of time when the construction stage starts, t_{cs} and t_{ce} are the instants of time when the concession period starts and ends, and n is the lifetime of the project.

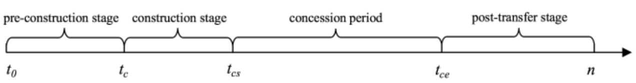

Fig. 1. A typical PPP project life cycle.

The cash inflow for a PPP project is the revenue received during the concession period for concessionaires or the post-transfer stage for governments, and the cash outflow mainly includes the construction cost, and operation and maintenance cost. If the theory of transaction economics is taken into consideration, the transaction cost, which means the cost incurred during an economic exchange, in a PPP project has been shown to be a non-ignorable number [12]. Thus, for a PPP project, the NPV for concessionaires and governments can be expressed as:

$$NPV_c = \sum_{t=t_{cs}}^{t_{ce}} (R_t - O\&M_t)/(1+r)^t - \sum_{t=t_0}^{t_{ce}} I_t/(1+r)^t - \sum_{t=t_c}^{t_{cs}} C_t/(1+r)^t \quad (1)$$

$$NPV_g = \sum_{t=t_{ce}}^{n} (R_t - O\&M_t)/(1+r)^t \quad (2)$$

where NPV_c indicates the net present value for concessionaires and NPV_g indicates the net present value for governments after the project is transferred from the concessionaire to the government, while R_t is the gross income from operating the project at year t, $O\&M_t$ is the operation and maintenance cost at year t, C_t is the yearly construction cost at year t, I_t indicates the transaction costs for concessionaries and r demonstrates the discount rate.

The discount rate used for NPV analysis should reflect the embodied risks in the project [13]. The weighted average cost of capital and the capital asset pricing model are the most commonly used methods for identifying the discount rates for PPP projects. However, it has been shown that traditional NPV analysis cannot fully reflect the project risk even though discounting the cash flow with a risk-adjusted discount rate and a Monte Carlo simulation of uncertainty variables should be a good supplement [14]. Brealey *et al.* [15] stated that the risk-free discount rate should be used in a Monte Carlo simulation since market risks have already been taken into consideration through assigning the probability distribution of the uncertain variables. Another method in reflecting project risk is to value the uncertain variables via geometric Brownian motion simulation. This method is particularly popular in estimating traffic volume in road-related projects, such as highways, bridges and tunnels [16], [17]. Like a Monte Carlo simulation, the risk-free discount rate should be used in geometric Brownian motion simulation, otherwise the risks are counted twice.

B. Real Option Application in PPP Projects

The NPV method cannot adequately reflect the values of the uncertainties in a project, which may lead to a fatal investment decision [18]. Even though some project parameters can be measured in a stochastic way, the values of the flexibilities embodied in the project are still waiting to be uncovered in NPV analysis. Smit and Trigeorgis [19] argued that the values of a project consist of the NPV, the flexible value and the strategic value, among which the project flexibility can be valued using real options analysis. In the options market, an option owner can choose to sell (put option) or buy (call option) the underlying asset before (American option) or only on the exercise date (European option) with a pre-specified strike price [20]. With real options, unlike financial options, their underlying assets are tangible assets. Over the past decade, the real options theory has been extended to evaluating the option value of government guarantees [10] and revenue caps [21].

Government guarantees are configured to help concessionaires to alleviate the market risk, especially when the project underperforms, and are also used as a method to attract private investors so the investment environment can become more positive [22]. Governments usually decide on a certain amount for the guarantee and write this into the contract. If the project revenue in a year is lower than the government guarantee, the government needs to provide compensation whose value equals the difference between the project's annual net income and the pre-specified guarantee threshold. If the guarantee value is viewed as the strike price of the guarantee option, then the option value of the government guarantee can be measured. Since the guarantee option as a real option can be exercised multiple times over a project lifetime, the guarantee option is measured as a multiple European put option in this paper. A European put option gives buyers the right, but not the obligation, to sell their underlying assets at the strike price on the expiry date of the option (the end of each year in this research). In other words, a European put option can be exercised once the asset price is lower than the strike price at the end of each year. Under this condition, the value of the put option is the difference between the market price and the strike price. In contrast, the put option will be worthless if the asset price is higher than the strike price.

Governments share more market risk through government guarantee agreements, but in turn they usually regulate a revenue cap, which is a cap on the annual profit of the concessionaires, to avoid suffering a heavy fiscal burden. If the net cash inflow in a year is higher than the revenue cap, the government could receive excess profit. Following this logic, the option value of the revenue cap is measured as a multiple European call option, as it can be exercised multiple times before the project ends once the asset price (the annual net income) is higher than the strike price (the value of the revenue cap) at the end of each year. A European call option gives buyers the right, but not the obligation, to buy their underlying assets at the strike price on the expiry date of the option.

III. Methodology

In this section, the methodology for deciding the optimal value of government guarantees and revenue caps is proposed, and contains the following steps: first, the optimal option value of a government guarantee is decided based on the local accounting standard, as well as the fair preferences held by the project parties. Second, since a government guarantee is being measured as a multiple European put option whose value cannot be decided directly, the specific value of the government guarantee is calculated through the backward induction of a binomial tree (i.e. the calculation starts from the end node of the binomial tree and moves to the starting point). Third, the revenue cap is valued as a supplement of the government guarantee in order to control the income gap between project parties and avoid overly lucrative conditions for concessionaires. Finally, a self-regulation process is created to control the project risk within an acceptable range when the government guarantee and the revenue cap work under the same contract package. The overall methodology flow chart can be seen in Fig. 2.

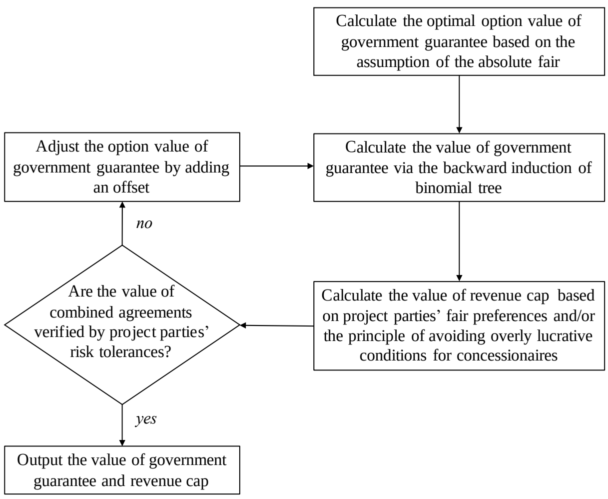

Fig. 2. Methodology flow chart.

A. Optimal Guarantee Option Decision Model

In order to calculate the value of a government guarantee using backward induction, the option value of the government guarantee needs to be decided first. According to the principle that concessionaires are willing to bid on a project when the profit to be gained from the project is higher than the expected minimum return on investment [23], the minimum option value of the government guarantee $\left(\overline{GG}_{min}\right)$ is decided by Eq. (3):

$$\overline{GG}_{min} = ROI_{min} - M(NPV_c) \tag{3}$$

where NPV_c is the number set involving all the values of NPV_c in m times geometric Brownian motion simulations. The mode of NPV_c, which is indicated as $M(NPV_c)$, is taken as the predicted value of NPV for concessionaires, since it is the most representative number in a non-normal distribution. ROI_{min} is the value of the minimum return on investment.

Governments provide a guarantee but do not want to lose capital liquidity, which is the reason that governments endeavour to locate the guarantee off the balance sheet. To achieve this, based on the PPP accounting standard suggested by the Australian Heads of Treasuries Accounting and Reporting Advisory Committee, concessionaires must bear the majority of risk in the project. Hence, a government guarantee cannot be granted without an upper limit, otherwise most of the risk would be transferred to the government side.

The maximum option value of a government guarantee should be the option value that minimises the difference in the risk borne by both parties while ensuring that the concessionaire holds most of the risk. Otherwise, an on-balance sheet record will be incurred. In mathematical language, the maximum option value of a government guarantee $\left(\overline{GG}_{max}\right)$ is decided by:

$$(P_c - P_g)\big|_{\overline{GG'}=\overline{GG}_{max}} \leq (P_c - P_g)\big|_{\overline{GG'}\neq\overline{GG}_{max}} \quad s.t.\ P_c - P_g > 0 \tag{4}$$

where $P_c = P(\overline{GG'} < ROI_{min} - NPV_c)$, $P_g = P\left(\overline{GG'} > NPV_g\right)$, P_c and P_g are the probabilities of suffering risk events in m times geometric Brownian motion simulations for concessionaires and governments respectively, and $\overline{GG'}$ is the independent variable in the solving process.

After deciding on the range of the option value of the government guarantee, the next step is to recognise the optimal option value of the government guarantee within the proposed interval. It can be learned from principal–agent theory that an excessive income gap between clients and agents is not conducive to the formation of a long-term partnership and the low-income party is prone to be 'envious' of the high-income earner, thereby affecting the normal operation of the incentive mechanism in partnership [24]. Thus, the methodology for deciding on the optimal option value of the government guarantee is proposed based on the fair preferences in relation to the project profits held by the project parties. The optimal option value of the government guarantee should be the one that minimises the income gap between the government and the concessionaire.

If the NPV for concessionaires is higher than the NPV for governments, the optimal option value of the government guarantee equals zero because governments do not want to waste financial resources to provide an additional guarantee:

$$if\ (NPV_c)_l \geq (NPV_g)_l, (\overline{GG'})_l = 0 \quad l = 1 \dots m \tag{5}$$

where l indicates the sequence number of geometric Brownian motion simulations, $(NPV_c)_l$ and $(NPV_g)_l$ are the simulated values of NPV for concessionaires and governments respectively in the number l sequence of m times geometric Brownian motion simulations, and $(\overline{GG'})_l$ is the value of $\overline{GG'}$ in the number l sequence of geometric Brownian motion simulations. The income gap cannot be controlled by the government guarantee in this case.

If the NPV for concessionaires is lower than the NPV for governments, flexible revenues for governments and concessionaires, which are the sum of the value of the NPV and the option value, need to be balanced. The best case is that there is no income gap between the project parties; this is called 'the absolute fair'. Mathematically, this can be indicated as:

$$if\ (NPV_c)_l < (NPV_g)_l, \left|NPV_g - \overline{GG'} - (NPV_c + \overline{GG'})\right|_l = k_l \to 0 \tag{6}$$

where k_l is a number indicating the income gap in the number l sequence of geometric Brownian motion simulations.

Then the value of $\overline{GG'}$ at each simulation process can be obtained from:

$$(\overline{GG'})_l = \lim_{k_l \to 0} [(NPV_g)_l - (NPV_c)_l \pm k_l]/2 \in \overline{GG'} \tag{7}$$

where $\overline{GG'}$ is the number set involving all the values of $\overline{GG'}$ that can minimise the income gap at each sequence of m times geometric Brownian motion simulations.

After obtaining all the option values of the government guarantee that can function to minimise the income gap between project parties, the optimal option value of the government guarantee can be decided following:

$$\overline{GG}_{opt} = M(\overline{\boldsymbol{GG'}}) \times P(NPV_c < NPV_g) \tag{8}$$
$$s.t. \overline{GG}_{opt} \in [\overline{GG}_{min}, \overline{GG}_{max}]$$

where $M(\overline{GG'})$ indicates the mode of the number set, $\overline{GG'}$. The constraint indicates that the optimal option value of the government guarantee should be located between the maximum and minimum option values of the government guarantee, otherwise the optimal option value cannot be found.

B. Government Guarantee Decision Model

As the government guarantee is here viewed as a multiple European put option, the government guarantee value, which is the strike price of the government guarantee option, cannot be measured directly. Thus, an indirect method based on the binomial tree model and the logic of backward induction is proposed.

The binomial tree model supposes that there are two directions for underlying assets to move: upward or downward. If the underlying assets move up, the assets' value in the next stage should be the initial value multiplied by the quotient (u), otherwise multiplied by the quotient (d), and the probability of moving up is expressed as (p). Only when the binomial tree and geometric Brownian motion share the same mean and variance in each step interval does the binomial tree model approximate to geometric Brownian motion. According to this, Cox *et al*. [25] proposed equations for binomial tree quotients: $u = e^{\sigma\sqrt{\Delta t}}$, $d = e^{-\sigma\sqrt{\Delta t}}$, $p = (e^{r\Delta t} - d)/(u - d)$, where σ is the volatility of the underlying assets, Δt is the step interval and r is the risk-free rate. Through the backward induction process, all the assets' value and the corresponding value can be generated at each node of the binomial tree.

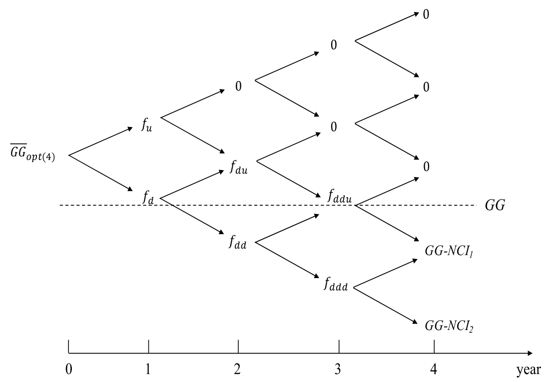

Fig. 3. The option value of a government guarantee in a binomial tree.

A four-year binomial tree, as shown in Fig. 3, is taken as an example to illustrate the process for setting the value of a government guarantee. Any branch below the government guarantee threshold will receive compensation from government. In a binomial tree, the option value of the government guarantee at the final stage is measured first and then the value is discounted back to the previous stage. What needs to be noted here is that the final stage for concessionaires should be the end of the concession period. Through repeating the discounting process, the value of the government guarantee, which is the only unknown variable in the process, can be obtained. The iterative discounting process for the government guarantee option exercised at the end of the fourth year, $\overline{GG}_{opt(4)}$, can be shown mathematically as:

$$f_{ddu} = e^{-r\Delta t}[p \times 0 + (1-p) \times (GG - NCI_1)] \tag{9}$$

$$f_{ddd} = e^{-r\Delta t}[p \times (GG - NCI_1) + (1-p) \times (GG - NCI_2)] \tag{10}$$

$$f_{dd} = e^{-r\Delta t}[p \times f_{ddu} + (1-p) \times f_{ddd}] \tag{11}$$

$$\overline{GG}_{opt(4)} = e^{-r\Delta t}[p \times f_u + (1-p) \times f_d] \tag{12}$$

where GG is the value of the government guarantee, NCI indicates the net cash inflow and f indicates the function of the option value, while r is the risk-free rate and Δt is the step interval in the binomial tree.

By repeating this process for each year in the binomial tree, the value of the government guarantee, as an independent variable embodied in Eq. (13), can be calculated.

$$\overline{GG}_{opt} = \sum_{j=t_{cs}}^{t_{ce}} \overline{GG}_{opt(j)} \quad (13)$$

where $\overline{GG}_{opt}$ is the optimal option value of the government guarantee calculated from Eq. (8).

C. Revenue Cap Decision Model

It can be noted that the proposed government guarantee can only minimise the income gap under the circumstances where concessionaires earn less than governments without any agreement being provided. As a result, the revenue cap as a supplement to the government guarantee mechanism is needed to control the income gap within a reasonable range when concessionaires earn more. Also, the revenue cap can be used to prevent concessionaires from earning excess money. The following section designs a revenue cap according to each purpose and tries to find the optimal value of the revenue cap that can control the income gap and overly lucrative conditions for concessionaires at the same time.

After taking the option value of the revenue cap into consideration, the income gap between the concessionaire and government is changed again and should be re-evaluated to meet their fair preferences. In order to revalue the income gap, the values of the NPV for concessionaires and governments with combined agreements need to be identified first. The government guarantee and the revenue cap work together, which forms the yearly revenue range for concessionaires. Namely, the annual net cash inflow for concessionaires must be located within this range. Therefore, the annual net cash inflow for concessionaires at sequence *l* geometric Brownian motion simulation (NCI_t^l) is indicated as:

$$NCI_t^l = \begin{cases} RC_1, & NCI_t \geq RC_1 \\ NCI_t, & GG < NCI_t < RC_1 \\ GG, & NCI_t \leq GG \end{cases} \qquad l = 1 \dots m \quad (14)$$

where NCI_t is an independent variable indicating the annual net cash inflow without boundaries defined by combined agreements and RC_1 is the value of the revenue cap designed to control the income gap. Since the value of the government guarantee has already been calculated, the value of the revenue cap can be decided directly without the need for the backward induction process on option value.

Based on the derived annual net income, the value of the NPV for concessionaires with combined agreements at sequence *l* geometric Brownian motion simulation $(NPV_c{}')_l$ is calculated through:

$$(NPV_c{}')_l = \sum_{t=t_{cs}}^{t_{ce}} NCI_t^l/(1+r)^t \quad (15)$$

Afterwards, the income gap at each simulation process (IG_l) is derived from:

$$IG_l = [(NPV_c{}')_l - (NPV_g)_l] \in IG \quad (16)$$

where IG is a number set, which stores all the value of the income gap in m times geometric Brownian motion simulations.

In order to control the income gap, the designed value of RC_1 should meet the prerequisite:

$$M(IG) \leq min\left(AIG_c, AIG_g\right) \quad (17)$$

where AIG_c and AIG_g are the maximum acceptable income gaps for contractors and public sectors respectively. The prerequisite is designed based on the principle that the revenue cap should be capable of controlling the income gap within the acceptable ranges for both parties.

All the values of the revenue cap that meet the conditions of Eq. (17) are stored in the number set, RC_1. In order to avoid damping down the concessionaires' keenness to invest, the maximum value of RC_1 is chosen as the optimal value:

$$RC_{opt1} = max(RC_1) \quad (18)$$

where RC_{opt1} is the optimal value of the revenue cap that can achieve the goal of controlling the income gap.

The revenue cap can also function to prevent overly lucrative conditions for concessionaires. If governments realise that the actual profit earned in a year surpasses the revenue cap threshold, the governments can claim the excess part of the revenue from the concessionaires. Under this condition, the annual net cash inflow for concessionaires at sequence *l* geometric Brownian motion simulation is indicated as:

$$NCI_t^l = \begin{cases} RC_2, & NCI_t \geq RC_2 \\ NCI_t, & GG < NCI_t < RC_2 \\ GG, & NCI_t \leq GG \end{cases} \quad l = 1 \dots m \quad (19)$$

where RC_2 is the value of the revenue cap designed to prevent overly lucrative conditions for concessionaires. The value of the NPV for concessionaires with combined agreements can be calculated via Eq. (15).

In order to prevent concessionaires from earning excess profits, the designed value of RC_2 should meet the prerequisite:

$$ROI_{min} \leq M(NPV_c') \leq ROI_{max} \quad (20)$$

where NPV_c' indicates the number set of NPV_c' in m times geometric Brownian motion simulations. The prerequisite illustrates that the profit for concessionaires should be less than the negotiated maximum return on investment signed off in the contract.

The optimal value of the revenue cap that can prevent overly lucrative conditions for concessionaires (RC_{opt2}) can be derived from:

$$RC_{opt2} = \max\left(RC_2\right) \quad (21)$$

where RC_2 indicates the number set of RC_2 that meets Eq. (20) in m times geometric Brownian motion simulations.

Finally, the value of the revenue cap that can prevent overly lucrative conditions for concessionaires while controlling the income gap between project parties can be found from:

$$RC_{opt3} = max\ (RC_1 \cap RC_2) \tag{22}$$

where, if the union of RC_1 and RC_2 returns to a null set, no suitable value of RC_{opt3} can be found, which means that overly lucrative conditions for concessionaires and the income gap cannot be controlled at the same time.

D. Self-adjustment on the Value of Combined Agreements

A revenue cap agreement interacting with a government guarantee may change the prior probabilities of suffering risk events for both parties. Hence, it is necessary to evaluate the project risk with consideration of the agreement as a whole. The best case is that the probabilities of suffering risk events for both parties are still within their risk tolerance ranges. For concessionaires, the risk event is that the profit earned is less than the minimum investment return. For governments, they need to bear the risk of suffering loss in the post-transfer stage. If a government guarantee and revenue cap decided through decision models lead to a risk occurrence rate that is higher than the risk capacity for any one of the project parties, the combined agreements will not play a role in risk control. Under this condition, the option value of the government guarantee needs to be revalued by adding or subtracting an offset to meet the project parties' risk tolerances, according to which the value of the revenue cap is revised. The new option value of the government guarantee can be found following:

$$\begin{gathered} \overline{GG}_{new} = \overline{GG}_{opt} \pm \Delta\overline{GG} \\ s.t.\ \overline{GG}_{new} \in \left[\overline{GG}_{min}, \overline{GG}_{max}\right] \end{gathered} \tag{23}$$

where $\Delta\overline{GG}$ is the value of the government guarantee offset. The constraint illustrates the principle that the revised option value of the government guarantee should be located between the $\overline{GG}_{min}$ and $\overline{GG}_{max}$ designed before. The revised value of the government guarantee is then calculated through the backward induction process of the binomial tree, following which the value of the revenue cap can be decided. Afterwards, the new government guarantee and revenue cap are verified again based on the project participants' risk tolerances. It should be noted that, as the revised option value of the government guarantee gradually deviates from the optimal value, the government guarantee's ability to control the income gap will decline with the increase in offset times. However, after stepping into the self-regulation process, project risk control should be the priority.

To summarise, when considering a government guarantee together with a revenue cap, the situation becomes complicated. As shown in Fig. 2, they work together to form a loop that can be seen as a self-regulation process for their values. Their values may need to be revised many times until they meet the project participants' risk requirements. The smaller the value of $\Delta\overline{GG}$, the greater the accuracy of the outcome of the self-regulation process that will be achieved.

IV. Numerical Example

A. Project Parameters

Before conducting the data analysis, the project parameters that are used in our designed models need to be clarified. Some of the project parameters show uncertainty, as they change with the trends of the market surroundings. These parameters are given based on the assumptions described below: the initial yearly traffic volume forecast for the project is $25{,}000 \times 1.2 \times 365 = 109{,}500{,}000$. The volatility of the future traffic volume is 12.5%. The operational cost is assumed to be 0.8 Australian dollars per car before the full charge for the toll road. Afterwards, the annual operating cost is set at 30% of the annual toll revenue as a more accurate standard [26]. The maintenance cost is 4 billion Australian dollars in the initial year with a 3% annual growth rate. The expected traffic growth rate 2.9%. Since the decision models calculate the cash flows based on the random traffic volume, the discount rate used for calculating the project NPV is the risk-free rate (2.6%), which equals the 10-year yield of Australian bonds. The underlying asset value used for the real options calculation equals the NPV value for the first operational year, which is 3.6×10^7 Australian dollars. The geometric Brownian motion model assesses the risks for a road-related project through assuming that the movement of traffic volumes follows a Markov process, as indicated in Eq. (24):

$$T_{t+\Delta t} = T_t \times e^{(\mu-\sigma^2/2)\Delta t+\sigma\varepsilon\sqrt{\Delta t}} \tag{24}$$

where Δt indicates the time interval, μ is the expected growth rate of the traffic volume during Δt, σ is the volatility of the uncertain variable during Δt, T_t is the current volume of the uncertain variable and ε follows a standardised normal distribution, $\varphi(0,1)$.

In addition to the uncertain parameters, the project itself has some inherent attributes: the construction period is 4 years and the project life is 60 years. The investment cost is 4.8 billion Australian dollars and it is assumed that there are no other transaction costs during the project life cycle. The minimum return rate on investment is 10% and the maximum return rate on investment allowed by governments is 20%. Both the contractor and public sector can tolerate a risk probability of less than 10% and it is assumed that they wish to control the income gap within 1 billion Australian dollars. Finally, based on the traffic volume in the initial year, the estimated annual net cash inflow should be around 3.0×10^7 Australian dollars in the operational stage.

B. Government Guarantee Determination

The trend of traffic volume has been widely viewed as a geometric Brownian motion. A 1000-times geometric Brownian motion simulation of traffic volume is generated to make the results of the data analysis more statistically significant and then the concessionaires' NPV for each geometric Brownian motion path is calculated. As seen in Fig. 4(a), the 1000-times NPV simulation for the concessionaires fluctuates mainly from 0.5×10^9 to 1.5×10^9 Australian dollars. However, it can also be observed that some of the simulated values of NPV are below the line indicating the value of the minimum return on investment for the concessionaires. In other words, the concessionaire suffer risk events at these excess-demand points.

The government guarantee is configured to protect the concessionaires from suffering financial risk. The number with the highest frequency in Fig. 4(b) is chosen as the value of the NPV for the concessionaire, which is 6.7×10^8 Australian dollars. As the NPV for the concessionaire is already higher than the minimum return on investment, the minimum option value of the government guarantee is set at zero. On the other hand, to make the guarantee off the balance sheet, the concessionaire need to bear the majority of the risk according to the Australia accounting rules for PPP projects, and therefore there should be a guarantee option cap for governments to prevent them from taking too much risk.

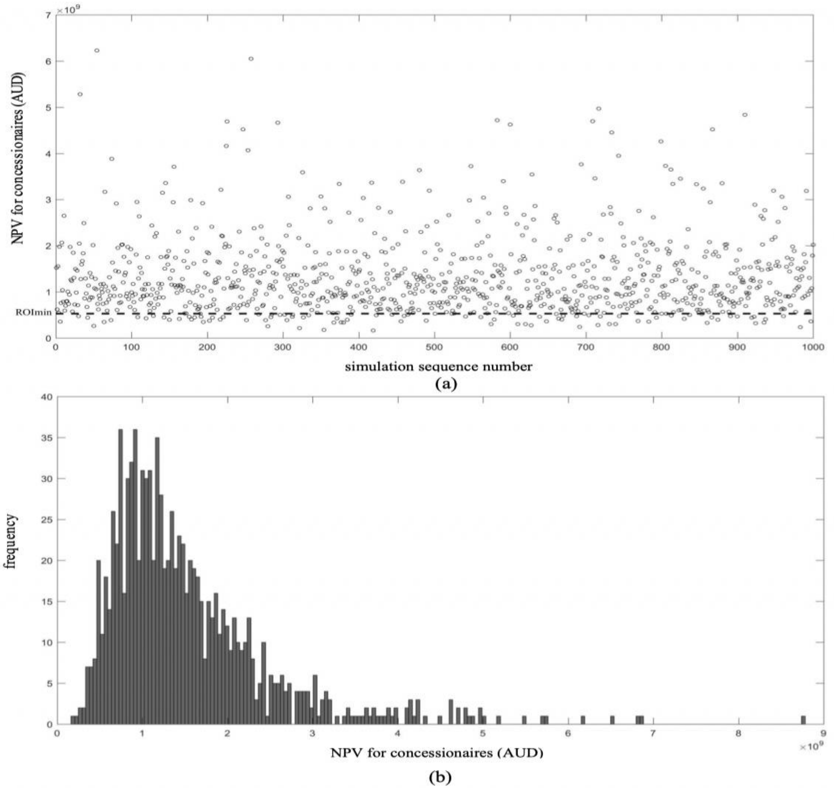

Fig. 4. (a) 1000-times *NPVc* simulation; (b) Frequency histogram of *NPVc*.

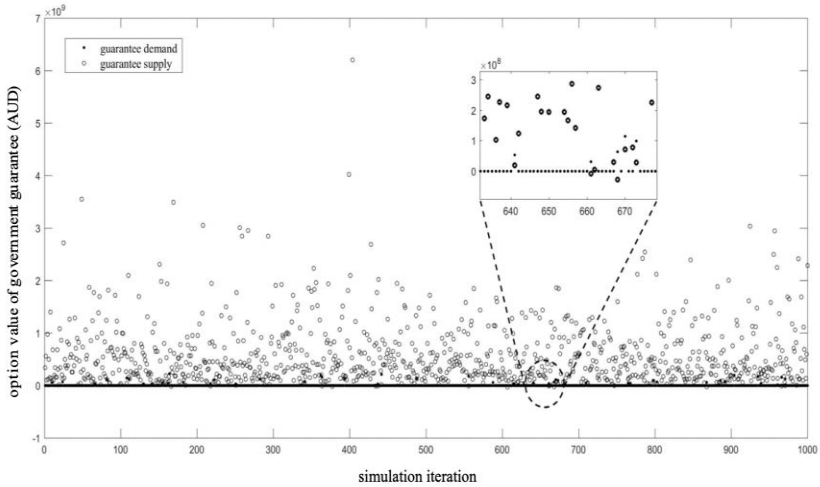

Fig. 5. Government guarantee supply-demand diagram.

To better illustrate how to determine the maximum option value of the government guarantee, Fig. 5 shows the option values of the government guarantees that the concessionaire need for each simulation process, which are indicated as black dots, and the corresponding option values of the government guarantees that governments can afford, which are indicated as hollow circles. If the designed maximum option value of the guarantee is higher than the affordable guarantees for governments in some simulation sequences, governments need to bear financial risk at these points. In contrast, if the designed maximum option value of the guarantee at some points is lower than the option value of the guarantees that concessionaires require, the concessionaire will suffer financial risk in these cases. Therefore, if the option value of the government guarantee as a constant is represented by a horizontal line in Fig. 5, the number of black dots above the line must be greater than the number of hollow circles below the line to make sure that concessionaires suffer more financial risk than governments. If there is more than one value of RG_{max} that satisfies the condition, the one that minimises the probabilities of difference in suffering risk between the project parties is the best choice. Following this logic, the option value of RG_{max} is decided to be 3.3×10^7 Australian dollars.

The optimal option value of the government guarantee that minimises the income gap is 2.1×10^7 Australian dollars, according to which the value of the government guarantee is calculated using the binomial tree model. Through backward induction, the threshold for triggering the government guarantee is set at 2.5×10^7 Australian dollars. With the designed optimal government guarantee threshold, the probabilities of suffering risk events for concessionaires and governments are 0% and 3.3% respectively. Both percentages are under the risk acceptance cap (10%) of the project parties. However, the return rate on investment for concessionaires solely with a government guarantee is up to 244% and the income gap is 3.9×10^8 Australian dollars (i.e. the concessionaire earns more), which is much higher than the tolerance ranges. Thus, it is necessary to design the revenue cap to cope with the overly lucrative conditions for the concessionaire and the excessive income gap.

C. Revenue Cap Determination and Self-adjustment Process

As introduced in the research background, governments define overly lucrative conditions as a maximum revenue return rate on investment of more than 20%. Following the methodology of preventing overly lucrative conditions for concessionaires, it can be noted that even though the value of the revenue cap equals the value of the government guarantee, all of the simulated values of the NPV are still more than 20% of the initial investment, which means that the overly lucrative conditions for concessionaires are beyond control. As there is no revenue cap with the value of the maximum return rate on investment (20%) given in the proposed project, the risk probabilities are the same as before. However, with the increase in the value of the maximum return rate on investment, as shown in Table I, the value of the revenue cap starts to grow and shows an increasing trend from 2.6×10^7 Australian dollars. The outcome tells governments that they cannot prevent concessionaires from earning excess money unless the maximum return on investment allowed by governments is increased to over 30%.

TABLE I: CHANGE OF REVENUE CAP (RC) WITH MAXIMUM RETURN RATE ON INVESTMENT (ROI_{max})

ROI_{max}	RC (million AUD)
20%	Unable to decide
30%	26.0
40%	27.9
50%	29.9
60%	31.9
70%	34.0
80%	36.1

If the revenue cap is designed to control the income gap between project parties within 1 billion Australian dollars, the value of the revenue cap rises to 1.3×10^8 Australian dollars. In order to verify the self-regulation process, the probabilities of suffering risk events with combined agreements are calculated, which are 0% and 2.8% for concessionaires and governments respectively. The thresholds of combined agreements are verified, as the project risks for both parties are within 10%.

To summarise, overly lucrative conditions for the concessionaire cannot be avoided unless the government allows a maximum return rate on investment that is higher than 30%. The value of the revenue cap which can control the overly lucrative conditions and the income gap at the same time for this project cannot be found. Nevertheless, when the value of the revenue cap is set at 1.3×10^8 Australian dollars, at least the income gap between the project participants is under control.

V. CONCLUSION

For a PPP project, governments design the value of the government guarantee to alleviate the inherent financial risks in the project, which encourages social capital to invest. In this research, a model for deciding the optimal value of government guarantees has been proposed. First, the determination of the lower limit of the government guarantee is based on the principle that the money earned by concessionaires should be at least higher than the minimum return on investment that they require. Second, considering that governments are not willing to bear high fiscal risks, the Australian PPP accounting standard is borrowed to make the granting guarantee off the balance sheet, based on which the upper limit of the government guarantee is decided. Finally, fair revenue allocation, which means minimising the income gap between the two parties in a PPP project, is the principle used to calculate the optimal value of the government guarantee. However, the government guarantee can only minimise the income gap under the circumstances that concessionaires earn less than governments without a government guarantee provided. In addition, governments cannot control overly lucrative conditions for concessionaires solely via government guarantees. Thus, governments design a revenue cap agreement that helps both to achieve fair revenue allocation and to minimise the income gap.

The proposed model is verified by a numerical example. Following the proposed determination process, the government guarantee threshold can be generated, according to which the value of the revenue cap is decided to control a large income gap and overly lucrative conditions for concessionaires respectively. The analysis outcome shows that with only a government guarantee, the risks for both parties can be controlled, but the issues of overly lucrative conditions for concessionaires and a large income gap still exist. After introducing a revenue cap, the income gap can be controlled within an acceptable range, but overly lucrative conditions for concessionaires still cannot be avoided unless the maximum return rate on investment allowed by governments is increased to over 30%. The research outcome also shows that overly lucrative conditions for concessionaires and an income gap cannot be handled simultaneously for this project. Finally, it should be noted that a government guarantee working with a revenue cap can change the probabilities of suffering risk for project participants, so a self-regulation process is designed to verify whether risk events are still under control. The designed values of combined agreements are verified through the self-adjustment process, but if they fail to pass the self-adjustment process, the option value of the government guarantee needs to be reconsidered by adding or subtracting an offset.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Hongyu Jin conceived and designed the analysis; Hongyu Jin performed the data analyses and wrote the manuscript; Chunlu Liu contributed to the conception of the study; Chunlu Liu helped perform the analysis with constructive discussions; all authors had approved the final version.

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