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Determination of Temperature Distribution in a Cylindrical Nuclear Fuel Rod – a Mathematical Approach

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***Abstract*—The steady state temperature distribution in a nuclear fuel rod has been discussed in this paper. Energy is released by fission within the nuclear fuel rod. This energy is then transferred by heat conduction to the surface of the fuel rod. From the surface of the nuclear fuel rod heat is transferred by convection to the coolant (water), which passes from the core to the external heat exchangers in which steam is generated to operate on a power cycle. The fuel elements are usually long cylindrical rods (which has been assumed in this paper) or rectangular plates of uranium (or thorium). The non-uniform heat generation has been employed in this paper. A cylindrical geometry with some assumptions has been considered. The geometry of the nuclear fuel rod is studied initially and then the symmetry considerations are carried out. The governing differential equation for the steady state condition with internal heat generation is rolled on. The numerical method of solution is used extensively in practical applications to determine the temperature distribution and heat flow in solids having complicated geometries, boundary conditions, and temperature-dependent thermal properties. A number of numerical schemes are available. In this paper the finite-difference method is used. The problem is then discretized and the numerical method of solutions is played out using finite difference method. Then the nature of the non-dimensional temperature distribution is observed and studied.**

***Index Terms*—Temperature distribution, Nuclear fuel rod, Steady-state Heat conduction, Internal heat generation, Finite difference method.**

I. Introduction

Energy is released by fission within the fuel rod and is transferred by heat Conduction to the surface of the fuel and through the cladding. [2] From the surface of the cladding heat is transferred by convection to the coolant, which passes from the core to the external heat exchangers in which steam is generated to operate on a power cycle. A nuclear fuel rod is used as the source of nuclear energy in a reactor. Most nuclear reactors are powered by fuel rods that contain two types of uranium: uranium-238 and uranium-235. The power generation process in a nuclear core is directly proportional to the fission rate of the fuel and the thermal neutron flux present._The thermal power produced by a reactor is directly related to the mass flow rate of the reactor coolant and the temperature difference across the core [4].

The fuel elements are usually long cylindrical rods or rectangular plates of uranium (or thorium) enclosed by cladding. The uranium may be in the pure metallic form, in the form of a compound such as uranium oxide, U02, or in the form of an alloy with another metal such as aluminum or zirconium [4]. The desirable properties of a fuel, which must be fissionable, are high thermal conductivity, good corrosion resistance, good mechanical strength at high temperatures and a high limiting temperature for operation.

The numerical method of solution is used extensively in practical applications to determine the temperature distribution and heat flow in solids having complicated geometries, boundary conditions, and temperature-dependent thermal properties. [3] A commonly used numerical scheme is the finite-difference method.

II. Considerations

- Consider the cross-section of a nuclear fuel rod as shown in Fig.
- Nuclear energy at a non-uniform rate of q" 'w/m^3 is generated in the rod.
- The surrounding coolant is at temperature, Ta and the heat transfer coefficient h is large.

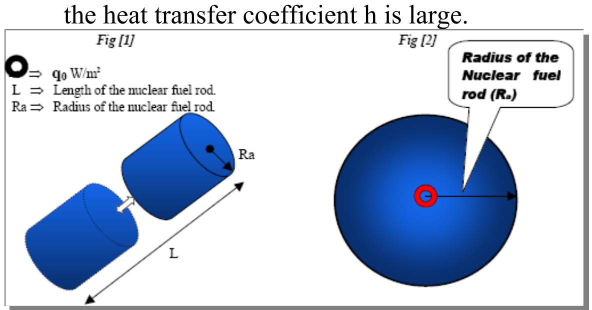

Figure1: Physical domain of the nuclear fuel rod:

A. *Internal Heat generation:*

The heat generation due to fission within a nuclear fuel rod is not uniform, and for a cylindrical fuel rod the heat generation is generally given by

$$q_g = q_o\,(1- \{r/R_a\}^2)\quad \text{W/m}^2 \qquad [1]$$

Where qo is the heat generation rate per unit volume at the centre (r = 0) and Ra is the outer radius of the solid fuel rod. Evidently qg is a function of position r, i.e., the radial distance from the axis of the rod [2].

For a steady state one-dimensional heat conduction in the radial direction

$$\mathbf{(d\,(r.\,dt / dr) / dr) + q_g.r / k = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots [2]}$$

$$\mathbf{(d\,(r.\,dt / dr) / dr) + q_o\,[1- \{r/R_a\}^2]\;r / k = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots [3]}$$

Upon integration

$$\mathbf{(r.\ dt / dr) + q_o [(r^2/2) - \{r^4/4R_a^2\}] / k = C_1 \ldots\ldots\ldots\ldots\ldots[4]}$$

$$\mathbf{(dt / dr) + q_o [(r/2) - \{r^3/4R_a^2\}] / k = C_1/r \ldots\ldots\ldots\ldots.[5]}$$

Integrating again,

$$\mathbf{T + q_o [(r^2/4) - \{r^4/16R_a^2\}] / k = C_1 \log_e r + C_2\ldots\ldots\ldots\ldots.[6]}$$

Invoking the boundary conditions,

$$\mathbf{At\ r = 0 \Rightarrow (dt / dr) = 0, T = T_{max}, then}$$

$$\mathbf{C_1 = 0 \ldots\ldots (From\ expr.\ 5)\ \&\ C_2 = T_{max} \ldots (From\ expr.6)}$$

Therefore equation 6 becomes,

$$\mathbf{T - T_{max} = - q_o [(r^2/4) - \{r^4/16R_a^2\}] / k \ldots\ldots\ldots\ldots\ldots[7]}$$

If T_w is the temperature at the outer surface (wall) of the rod i.e., at $r = R_a$, then

$$T_w - T_{max} = - 3\ q_o R_a^2/ 16\ k \ldots\ldots\ldots[8]$$

The heat flow at the surface of the fuel rod is,

$$Q = - k\ A\ (dt/dr).\ @\ r = R_a$$
$$Q = - k\ A\ [- q_o [(r/2) - \{r^3/4R_a^2\}] / k].\ @\ r = R_a$$
$$Q = - k\ A\ [- q_o [(R_a /2) - \{ R_a^3/4R_a^2\}] / k]$$
$$Q = q_o\ A\ R_a/ 4 \ldots\ldots\ldots\ldots.[9]$$

Under steady state conditions, this heat would be converted from the outside surface of the rod.

$$(q_o\ A\ R_a/ 4) = h\ A\ (T_w - T_a)$$
$$T_w = T_a + (q_o\ R_a /4h) \ldots\ldots\ldots.. [10]$$

Where h is the convective heat transfer coefficient and T_a is the ambient temperature. Therefore from equations 8 and 10, we get

$$T_{max} - T_a = (q_o\ R_a/ 4) [(3\ R_a /4k) – (1/h)] \ldots\ldots\ldots\ldots\ldots[11]$$

III. Governing Differential Equation

When a heat conduction problem is solved exactly by an analytical method, the resulting solution satisfies the governing differential equation at every point in the region as well as at the boundaries. However, when the problem is solved by a numerical method, such as finite differences, the differential equation is satisfied only at a selected number of discrete nodes within the region [3]. Therefore, the starting point in an analysis by the finite-difference method is the finite-difference representation of the heat conduction equation and its boundary conditions.

The governing non-dimensional energy equation for the steady state one dimensional radial heat conduction with non-uniform internal heat generation for a cylindrical nuclear fuel rod is given as [5]

$$\mathbf{(\partial^2\theta/\partial R^2) + (\partial\theta/\partial R) / R + 1 = 0 \quad \ldots\ [12]}$$

Where,

$$\theta = [T - T_a] / [(q_o\ R_a/ 4) [(3\ R_a /4k) – (1/h)]]$$
$$R = (r / R_a)$$

A. Boundary Conditions

The non-dimensional boundary conditions are @ $R= 0 \Rightarrow \partial\theta/\partial R = 0$ & @ $R = 1 \Rightarrow \theta = 0$

B. Symmetry Considerations

A close look at the physics of the problem reveals that the problem is geometrically and thermally symmetric [1]. Therefore, from the temperature distribution in any radial of the physical domain, by mirror imaging one can get the solution for the entire region. The figure shows the computational

domain in radial direction. The use of the symmetry enables the numerical analyst to obtain the solution much faster as the number of grid points is reduced greatly.

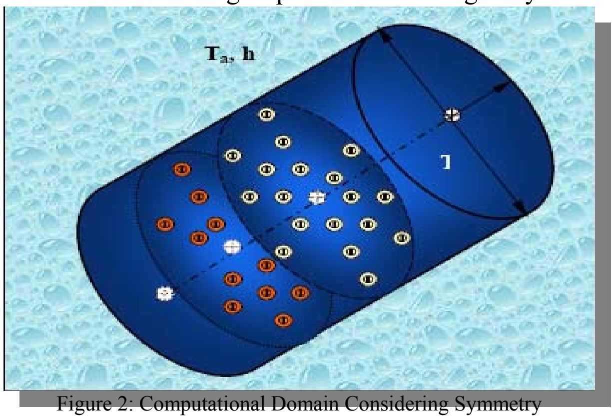

Figure 2: Computational Domain Considering Symmetry

IV. Discretization

The computational domain including the notations for the interior grid points is shown in the fig [4]. Equation 12 is discretized using central difference for $(\partial^2\theta/\partial R^2)$ and $(\partial\theta/\partial R)$ at any interior grid point‘ i ‘ as follows,

$$[(\theta_{i-1} -2\theta_i + \theta_{i+1})/(\Delta R)^2]+ [(\theta_{i-1} - \theta_{i+1}) / 2.R.\ \Delta R] +1= 0\ldots\ldots\ldots.. [13]$$

$$[[(\theta_{i-1} -2\theta_i + \theta_{i+1}).\ (2R) + (\theta_{i-1} - \theta_{i+1}).\ \Delta R + (\Delta R)^2.\ (2R)]/ (2R)] = 0\ldots\ldots. [14]$$

$$(\theta_{i-1} -2\theta_i + \theta_{i+1}).\ (2R) + (\theta_{i-1} - \theta_{i+1}).\ \Delta R + (\Delta R)^2.\ (2R) = 0 \ldots\ldots. [15]$$

A. Handling of corner points

Handling of corner points require special attention as it depends upon the conditions avail. If the surface has Dirichlet condition then there is no problem because the corner point can be assumed to be having a specified temperature. But, if the surface has Robbins/Neumann condition, then the corner point has to be handled separately. With respect to the present problem, there are two corner points. Of these, one point is exposed to Dirichlet condition and the other to Neumann condition.

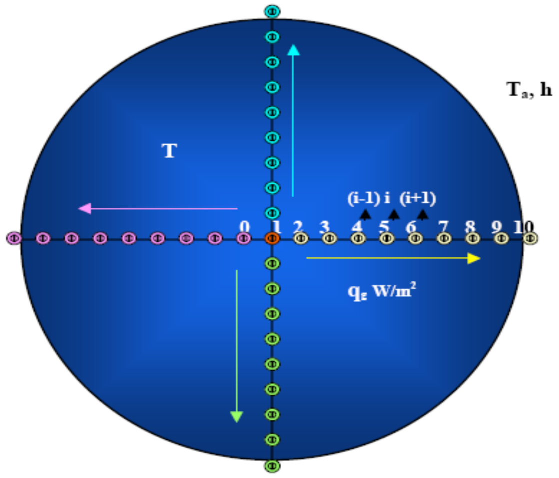

Figure 3: Computational Domain Considering Symmetry in cross-section

B. Treatment of the condition at the centre i.e. at R = 0.

A close look at eq.12 which is the diacretized form of the GDE reveals that at the centre i.e. at R=0, the second term on the RHS of the GDE will become infinity which is certainly not acceptable.

i.e. The second term on the RHS of the GDE can be written as $(\partial\theta/\partial R)/R$. At R=0, $\partial\theta/\partial R = 0$ from the second boundary condition. Therefore the term $(\partial\theta/\partial R)/R$ will give rise to 0/0 condition.

However, this difficulty can be alleviated by making use of the L'Hospital's rule.

If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0.$

and $\lim_{x\to a} (g'(x)/f'(x)) = L.$

then $\lim_{x\to a} (g(x)/f(x)) = L.$

Therefore, invoking L'Hospital's rule to the centre condition, we obtain,

$$[(\partial\theta/\partial R)/R] = [\partial(\partial\theta/\partial R)/\partial R] / [\partial(R)/\partial R]$$
$$[(\partial^2\theta/\partial R^2)/1] = \partial^2\theta/\partial R^2$$

Hence at R=0, putting $[(\partial\theta/\partial R)/R] = (\partial^2\theta/\partial R^2)$, eq.12 becomes,

$$2[\partial^2\theta/\partial R^2] + 1 = 0$$
$$2[(\theta_{i-1} - 2\theta_i + \theta_{i+1})/(\Delta R)^2] + 1 = 0 \text{ .... [16]}$$

At the centre,

i =1, i-1=0, i+1=2.

Using mirror-image technique at the centre R=0,

i-1=0 i=1 i+1=2

Therefore eq .16 becomes,

$$2[(\theta_{i-1} - 2\theta_i + \theta_{i+1})/(\Delta R)^2] + 1 = 0$$
$$2[(\theta_0 - 2\theta_1 + \theta_2)/(\Delta R)^2] + 1 = 0$$
$$2[(2\theta_2 - 2\theta_1)/(\Delta R)^2] + 1 = 0$$
$$4[(\theta_2 - \theta_1)/(\Delta R)^2] + 1 = 0$$
$$4(\theta_2 - \theta_1) + (\Delta R)^2 = 0 \text{ .... [17]}$$

C. Treatment of the condition at the outer boundary i.e. at R = 1.

The outer boundary is maintained at temperature Ta, the temperature of the surrounding fluid, assuming a large heat transfer coefficient h (i.e.) Tw =Ta.

At R=1, $\theta = 0$ …. [Second boundary condition]

V. Method Of Solutions

Let us consider an example in which $\Delta R = (1/9)$. Therefore, the number of nodal points is ten (i.e. from 1 to 10) and the numbers of unknown temperatures are nine (i.e. from 1 to 9). Since T=Ta at the outer boundary, the value θ of is zero at the nodal point 10.

Recalling the eq.15,

$$(\theta_{i-1} - 2\theta_i + \theta_{i+1}).(2R) + (\theta_{i-1} - \theta_{i+1}).\Delta R + (\Delta R)^2.(2R) = 0$$

$$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})(\Delta R/2R) = (\Delta R)^2 \text{....... [18]}$$

- ***At Nodal Point*** 1

R=0 & i = 1

$4(\theta_2 - \theta_1) + (\Delta R)^2 = 0$..Recalling eq. 17

$\mathbf{4\theta_1 - 4\theta_2 = (1/9)^2}$

- ***At Nodal Point*** 2

R = (1/9) & i = 2

$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})(\Delta R/2R) = (\Delta R)^2$ …. Recalling eq. 18

$\mathbf{2\theta_2 - (3/2)\,\theta_1 - (1/2)\,\theta_3 = (1/9)^2}$

- ***At Nodal Point*** 3

R = (2/9) & i = 3

$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})(\Delta R/2R) = (\Delta R)^2$ …. Recalling eq. 18

$\mathbf{2\theta_3 - (5/4)\,\theta_2 - (3/4)\,\theta_4 = (1/9)^2}$

- ***At Nodal Point*** 4

R = (3/9) & i = 4

$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})(\Delta R/2R) = (\Delta R)^2$ …. Recalling eq. 18

$\mathbf{2\theta_4 - (7/6)\,\theta_3 - (5/6)\,\theta_5 = (1/9)^2}$

- *At Nodal Point* 5

R = (4/9) & i = 5

$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})(\Delta R/2R) = (\Delta R)^2$ …. Recalling eq. 18

$\mathbf{2\theta_5 - (9/8)\,\theta_4 - (7/8)\,\theta_6 = (1/9)^2}$

- *At Nodal Point* 6

R = (5/9) & i = 6

$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})(\Delta R/2R) = (\Delta R)^2$ …. Recalling eq. 18

$\mathbf{2\theta_6 - (11/10)\,\theta_5 - (9/10)\,\theta_7 = (1/9)^2}$

- *At Nodal Point* 7

R = (6/9) & i = 7

$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})(\Delta R/2R) = (\Delta R)^2$ …. Recalling eq. 18

$$\mathbf{2\theta_7 - (13/12)\ \theta_6 - (11/12)\ \theta_8 = (1/9)^2}$$

- *At Nodal Point* 8

$R = (7/9)$ & $i = 8$

$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})\ (\Delta R/2R) = (\Delta R)^2$ ….

Recalling eq. 18

$$\mathbf{2\theta_8 - (15/14)\ \theta_7 - (13/14)\ \theta_9 = (1/9)^2}$$

- *At Nodal Point* 9

$R = (8/9)$ & $i = 9$

$(2\theta_i - \theta_{i-1} - \theta_{i+1}) + (\theta_{i+1} - \theta_{i-1})\ (\Delta R/2R) = (\Delta R)^2$ ….

Recalling eq. 18

$$\mathbf{2\theta_9 - (17/16)\ \theta_8 - (15/16)\ \theta_{10} = (1/9)^2}$$

- *At Nodal Point* 10

$R = (9/9) = 1$ & $i = 10$

Recalling the second boundary condition

$@\ R = 1 \Rightarrow \theta = 0$

$\mathbf{\theta_{10} = 0.}$

These are the 9 equations to be solved to find the 9 unknown temperatures. These 9 equations can be written in the matrix form and can be solved through various methods like Gaussian elimination, Gauss-seidel iteration, etc. As those analytical approaches are too lengthy if the nodal points are more, computer programming is mostly preferred. Here matlab program is used to solve the above matrix. The results thus obtained are shown as the graphical representation in the next chapter.

VI. Graphical Representations

A graph is plotted between the θ value & the nodal points along the radial direction to show how the temperature varies from the mid point i.e. from the center towards outer surface. As θ value is

directly proportional to $[T - T_a]$ or $[T - T_w]$, the maximum the value of θ, the maximum the value of T is and vise-versa. From the graph it is clear that the value of θ decreases from the nodal point 1 to 9. This shows that the non-dimensional temperature decreases from the center of the nuclear fuel rod towards the outer surface. As the θ value is directly proportional to $[T - T_a]$ or $[T - T_w]$, it is clear from the graph, the temperature also decreases in the same fashion as θ. We know,

$\boldsymbol{\theta} = [T - T_a] / [(q_o R_a/ 4) [(3 R_a /4k) - (1/h)]]$ **(or)** $\boldsymbol{\theta} = [T - T_w] / [(q_o R_a/ 4) (3 R_a /4k)]$… (If h >>>>1)

Thus by using the above formula for a given nuclear fuel rod, the temperature at the required grid point can be determined provided the values other than T are known.

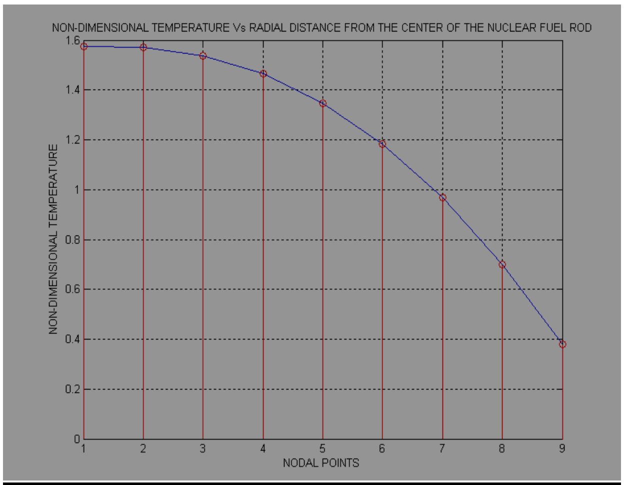

Figure 4: Non-dimensional temperature distribution along the nodal points from the center.

The governing differential equation for the steady state heat conduction with internal heat generation is considered. The non-uniformity of the internal heat generation with respect to the radial distance has also been taken in to account. Here the cylindrical nuclear fuel rod is encountered by symmetry considerations, governing differential equations, and boundary conditions. Then discretization is carried out followed by the method of solutions. Finally, the non-dimensional temperatures are determined.

VII. Conclusions

The graphical representation between the non-dimensional temperature and the radial distance from the center of the nuclear fuel rod unveils that the temperature is maximum at the center and minimum at the outer diameter. This shows the heat is transferred from the nuclear fuel rod to the surrounding coolant according to the second law of thermodynamics. Thus from the surface of the nuclear fuel rod, heat is transferred by convection to the coolant, which passes from the core to the external heat exchangers in which steam is generated to operate on a power cycle. In this paper the temperature distribution for the nuclear fuel rod without cladding has been performed. Hence the same problem can be encouraged with cladding which is directed as a scope for future work. This paper explores a totally different type of approach to determine the temperature distribution and can be applicable for the problems of same motive.

NOMENCLATURE

T,t Temperature (K)

q Heat Flux (W/m^2)

R,r Radius of the nuclear fuel rod (m)

h Heat transfer coefficient (W/m^2K)

k Conductivity (W/mK)

L Length of the nuclear fuel rod (m)

- Greek letters

θ Non-dimensional temperature.

Δ Difference.

- Subscripts

a Atmospheric / fuel rod

g Internal generation

o Internal generation at the center

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