Flight Trajectory of a Golf Ball for a Realistic Game

Seongmin Baek and Myunggyu Kim

***Abstract*—This paper suggests a new flight model for a realistic golf game simulation. The forces at work in a ball during flight include lift, drag, and gravity. A golf ball flying at a high spinning speed has a different lift and drag according to its spin rate and Reynolds number. Also, the drag force of a ball is largely different according to the size, depth, and number of dimples in the ball. However, these differences are not reflected in golf game simulations. This paper suggests a method for changing the simulated flight distance of a golf ball in accordance with its drag based on changes in Reynolds number and dimple characteristics. Also, since the flight distance of a ball changes in real world depending on the temperature, humidity, and altitude, these changes should be reflected in realistic games as well. When the temperature, humidity, and altitude are given, the density and pressure of air in the virtual environment are calculated, and through these calculated values, the flight distance of the ball can be changed. Finally, the effect of wind should be considered. Usually, in games, the wind effect is handled using a constant term. However, the strength of a real wind changes following its height. By applying a function reflecting this change, the strength of the wind is reflected differently according to the elevation of the ball. Through the method suggested in this paper, we can calculate a realistic flight trajectory for a simulated golf ball.**

***Index Terms*—Spinning ball, flying trajectory, aerodynamic characteristics.**

I. INTRODUCTION

When playing a simulated golf game in a virtual environment such as screen golf, it is important that the flight distance of the golf ball be predicted and marked exactly on the screen. To accurately simulate a flight trajectory, the forces at work in the spinning ball should be calculated precisely. Many researches on the forces at work in ball flight have been carried out [1], but it is not easy to apply the varying elements to virtual games. Bearman and Harvey [2] conducted a comprehensive study of a golf ball in a wind tunnel and found that the C_L increased in proportion to the spin rate, and that the C_D was highly dependent on Reynolds number (Re). As the Reynolds number increases, the C_D suddenly decreases to a minimum value, and later begins to slowly increase. However, they did not present a model for calculating coefficients. Smits and Smith [3] tried to come up with a C_L and C_D through a wind tunnel test and suggested the new aerodynamic model to calculate these values. However, this model did not include the dimple effects of a golf ball.

Borg [4] suggested a method to seek the trajectory of a smooth-surfaced ball. However, he did not consider the dimples in a real golf ball. As a golf ball with dimples has a smaller drag force and a greater lift than a smooth ball, it flies further.

The drag coefficient changes in accordance with the size, depth, number of dimples in a ball. Ting [5] demonstrated that as the size of the dimples increases, the drag coefficient decreases. He also proved experimentally that as the depth of the dimples increases, its drag coefficient decreases, and that at a certain depth, the drag coefficient increases again. Aoki [6] proved experimentally that when the number of dimples increases, its C_D reaches a minimum value in terms of lower Reynolds number.

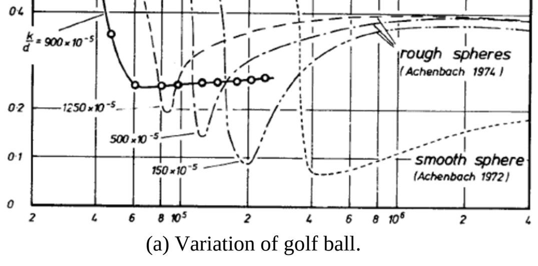
A ball in flight also varies based on the weather conditions. For example, a ball flies further as the temperature rises and the humidity lowers. A ball hit at high land flies further away. That is, as the density of air lowers, a ball flies further. Thus, the calculation of air density is essential. Wind is also an important element in the flight trajectory of a ball. The direction and strength of the wind alters the speed of the ball, changing its drag and lift coefficients. Usually, screen golf games use wind speed as a constant term, but real wind strength varies according to its elevation [7].

As mentioned above, when considering drag, lift, condition of the air, and the effect of dimples, calculating the correct flight trajectory is possible. Based on the air conditions and dimple effect, the flight trajectory can change. As a result, screen golf games given correct and varying data enhance their feeling of realism and enjoyment.

II. LIFT COEFFICIENT

Lift force formed in the direction vertical to the ball movement is expressed as in Equation (1). Here, C_L is the lift coefficient, ρ is the air density, A is the cross section of a ball, and v_b is the ball speed.

$$F_L = \frac{1}{2} \cdot C_L \cdot \rho \cdot A \cdot v_b^2 \tag{1}$$



(a) Variation of golf ball.

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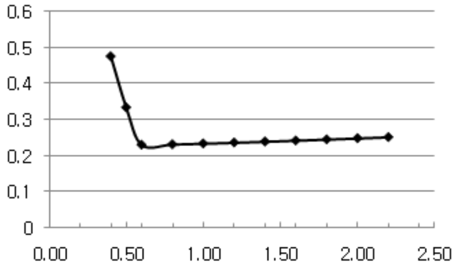
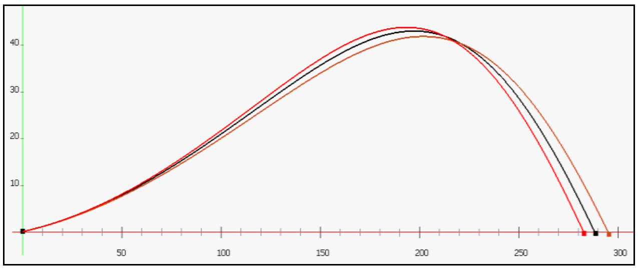


(b) Our Drag Coefficient Model.
Fig. 1. Drag coefficient for different Reynolds numbers.

The lift coefficient determines how much synergistic effect a spinning ball obtains and is proportional to its spin rate. The lift coefficient is expressed as in Equation (2) [8].

$$C_L = -0.05 + \sqrt{0.0025 + 0.36R \cdot \frac{|\omega|}{|v|}} \qquad (2)$$



[Red: 10˚C, 0%], [Black: 20˚C, 0%], [Brown: 30˚C, 70%]
Fig. 2. Change of Temperature and humidity.

III. DRAG COEFFICIENT

Drag force is similar with lift force and is expressed as in Equation (3).

$$F_D = \frac{1}{2} \cdot C_D \cdot \rho \cdot A \cdot v_b^2 \qquad (3)$$

The drag coefficient determines how much air resistance a ball in flight receives. In most games, the drag coefficient is treated as a constant term (0.21 ~ 0.25). However, the drag coefficient has a deep connection with the Reynolds number. Usually, the drag coefficient sharply decreases at a critical regime (4x104 to 6x104), and from that critical regime, it gradually increases. We suggest a new function for determining the drag coefficient based on the results of Bearman's experiment [2] (Equation (4)).

Fig. 1(a) shows the relationship between Reynolds number and drag coefficient. As shown in Fig. 1(b), a change in drag coefficient caused by the Reynolds number is similar with Bearman's results.

$$\begin{aligned}
C_D &= 0.2136 \cdot (-2.1 \cdot \exp(-0.12 \cdot (D(\mathrm{Re}) + S + 0.35)) \\
&\qquad + 8.9 \cdot \exp(-0.22 \cdot (D(\mathrm{Re}) + 0.35)) \\
D(\mathrm{Re}) &= K_1 + C_{d1}K_1 - 1 \qquad\qquad (\mathrm{Re} < \mathrm{Re}_{cr}) \\
&= K_2 + C_{d1}K_2 - 0.0225C_{d2} - 1 \qquad (otherwise) \\
\mathrm{Re}_{cr} &= 0.6 \times 10^5 \\
K_1 &= \mathrm{Re} \cdot C_{d3}\ , \quad K_2 = \mathrm{Re} \cdot C_{d3} - C_{d2} \\
C_{d1} &= 0.25\ , \quad C_{d2} = (\mathrm{Re} - \mathrm{Re}_{cr}) \cdot C_{d3}\ , \quad C_{d3} = 0.1 \times 10^{-3} \\
Range &: 0.4 \times 10^5 \quad to \quad 2.2 \times 10^5
\end{aligned} \qquad (4)$$

IV. ATMOSPHERE

A. Density of Air

The flight distance of a golf ball is related with the air density because the lift and drag forces acting on a spinning ball are proportional to it. However, entering the air density as a parameter in a game is not intuitive because user can't understand it. Therefore, the air density should be expressed using temperature, humidity, and altitude, which are factors a player can easily understand. Equation (5) shows how air density is calculated [9]

$$\rho = 3.48374 \cdot P \cdot Z^{-1} \cdot T^{-1} \cdot (1 - 0.378x_v) \qquad (5)$$

Here, **P** is pressure; **T** is temperature; x_v is the function of humidity ($0 \le x_v \le 1$), pressure, and temperature; and Z is the function of x_v, temperature, and pressure.

Pressure is expressed using a function that changes according to altitude (h), as in Equation (6).

$$P = P_b \cdot \exp(-gM \cdot h \cdot R^{-1} \cdot T_b^{-1}) \qquad (6)$$

The use of parameters such as temperature, humidity, altitude, and so on is more intuitive, and air density can be adjusted through changes in these parameters. As air density changes, both drag and lift change, which influences the flight trajectory. Fig. 2 shows the flight distance of a ball when the temperature and humidity are changed, just like on a real field.

B. Wind

Wind is one of main elements that cause a change in the flight trajectory of a ball. In virtual games, the effect of wind is dealt with as a constant term, but the strength of real wind is a function that changes based on its height (Figure 3). This paper calculates wind speed according to the height of a ball using Equation (7) [7].

$$\begin{aligned} v_2 / v_1 &= (z_2 / z_1)^n \\ n &= 0.37 - 0.0881 \cdot ln(v_1) \end{aligned} \qquad (7)$$

Equation (7) indicates that when the wind speed measured at predetermined height (z_1) is v_1, the wind speed at a certain height (z_2) is v_2. To apply the effects of wind when calculating drag and lift, the speed of the ball, $\boldsymbol{v_b}$, should be replaced with $\boldsymbol{v_b - v_2}$.

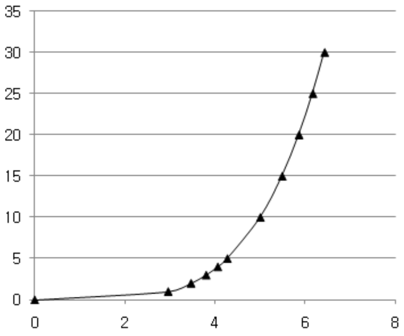


Fig. 3. Example of wind speed (5 m/s at 10 m height).

V. DIMPLE EFFECTS

A ball with dimples flies further than a smooth ball.

However, the flight distances of balls differ according to their dimple characteristics, so it is difficult for them to be simulated exactly. This paper approximates the function of the size, depth, and number of dimples in order to apply it to its equation. Thanks to this approximation, the flight trajectory of a ball can change according to these dimple characteristics.

A. Dimple Size

The size of a dimple is bigger, its drag coefficient diminishes. When the size reaches a certain point ($c/d \approx 0.08$), the drag coefficient reaches a regular constant value [5]. Equation (8) shows a model using these characteristics.

$$
\begin{aligned}
C_{D\_size} &= R_1 \cdot (-3.125 \cdot (c/d) + 0.25 \qquad (c/d < 0.08) \\
&= 0 \qquad\qquad\qquad\qquad\qquad\quad (c/d \geq 0.08)
\end{aligned}
\tag{8}
$$

Here, c/d indicates the ratio of dimple diameter to the ball diameter. Figure 5 shows the values calculated using this equation. We calculated C_{D_size} following the c/d of the dimples and added it to the drag coefficient. At that time, R_1 indicates a weight ratio of C_{D_size}. The flight trajectories of a ball according to changing of dimple size are shown in Figure 6.

B. Dimple Depth

The deeper the dimple is, the smaller its drag coefficient becomes. The ratio of dimple depth to ball diameter (k/d) is at a minimum in about 0.005. As the depth increases, the drag coefficient begins to increase [5].

The function for the depth of a dimple can be calculated using the interpolation of two polynomial expressions (Y_1 and Y_2) as in Equation (9). As shown in Figure 7, this is similar with the value given in Ting's experiment. The value of C_{D_depth} obtained through Equation (9) is added to the drag coefficient. R_2 is a weight ratio of C_{D_depth}. Figure 8 shows the flight distance of a ball when the depth of dimples is changed.

$$
\begin{aligned}
&Y_1 = A_1 X_1^2 + B_1 X_1 \ , \ \ (A_1 = 0.9295,\ B_1 = 0.06474) \\
&Y_2 = A_2 X_2^2 + B_2 X_2 \ , (A_2 = 0.2326,\ B_2 = -0.00263) \\
&C_{D\_depth} = R_2 \cdot (Y_1 \cdot K_1 + Y_2 \cdot (1 - K_1)) \\
&K_1 = Norm(pow((k/d - 0.5)^2, 0.1) \cdot SIGN(k/d - 0.5)
\end{aligned}
\tag{9}
$$

C. Dimple Number

A change in drag coefficient based on the number of dimples is quite complicated. According to Aoki's experiment [6], as the number of dimples increases, the drag coefficient reaches a minimum value at a lower Reynolds number. However, as the number of dimples moves beyond a certain number (about 330 dimples), the drag coefficient shows a similar effect (Fig. 9(a)).

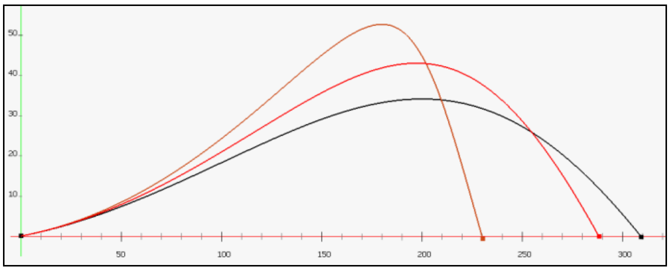
In order to apply such an effect, in the previous equation for drag coefficient, the value of K_1 and K_2 is changed as in Equation (10).

$$K_1 = K_1 - D_n \ , \ \ K_2 = K_2 - D_n \tag{10}$$

Here, D_n is given as shown in Table I (range: 104 to 504). The coefficient for the random number of dimples is determined using the interpolation of the given values. Changes in drag coefficient based on the number of dimples are shown in Fig. 9(b). As the number of dimples increases, the drag coefficient has a minimum value at a lower Reynolds number. When D_n is 504, the section with a minimum value is similar with that of 328, but later, the value of C_D increases somewhat. Fig. 10 shows the flight distance of a ball when the number of dimples is changed.

TABLE I: COEFFICIENTS FOR THE NUMBER OF DIMPLES

Num.	0.4	0.5	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	$\times 10^5$
104	0	1.25	2	1.9	1.2	-0.5	-0.55	-0.6	-0.65	-0.7	-0.75	
184	0	1.25	2	1.5	0	-0.4	-0.45	-0.5	-0.55	-0.6	-0.65	
328	0	0	0	0	0	0	0	0	0	0	0	
504	0	0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	



[Black: 5m/s], [Red: 0m/s], [Brown: -5m/s, headwind]

Fig. 4. Flight distance by wind effect.

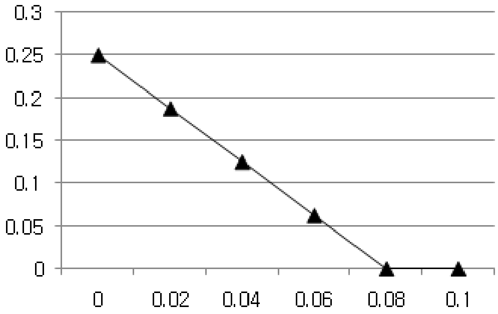
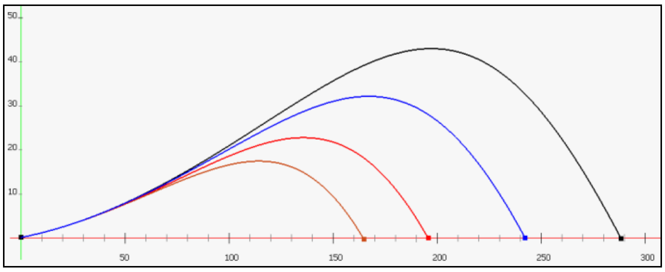


Fig. 5. Drag coefficient *vs*. *c*/*d*.



[Black: 0.08], [Blue: 0.06], [Red: 0.03], [Brown: 0.0, smooth ball]

Fig. 6. Dimple size (*c*/*d*) *vs*. flight distance.

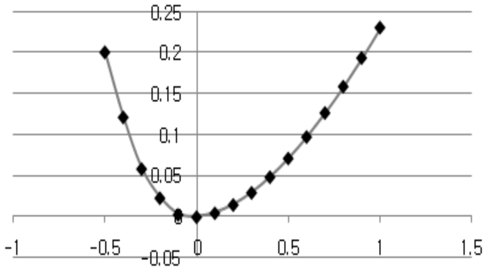
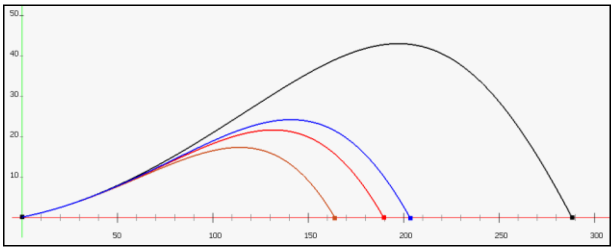
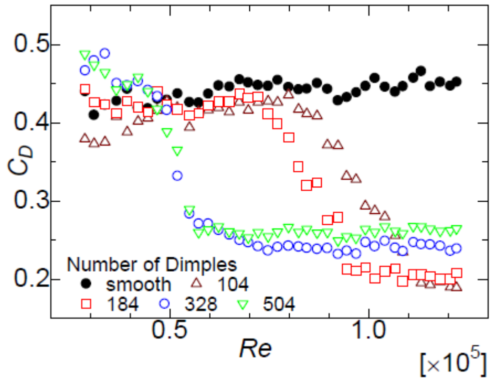


Fig. 7. Drag coefficient *vs*. k/d

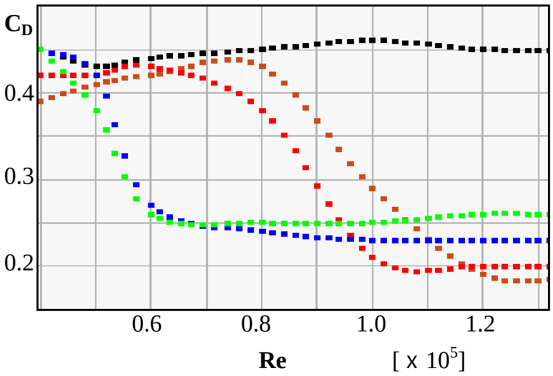


[Black: 0.005], [Blue: 0.012], [Red: 0.008], [Brown: 0.0, smooth ball]

Fig. 8. Dimple depth (*k*/*d*) *vs.* flight distance.

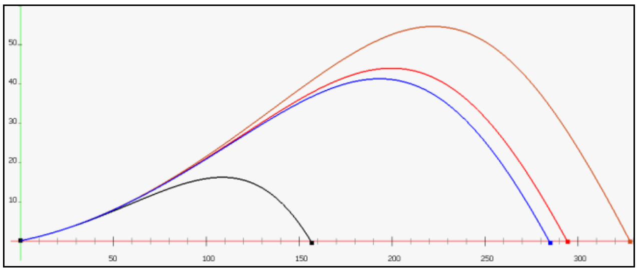


(a) Change of CD to Dimple Number



(b) Our Model for Dimple Number: Black (without dimple), Brown (104), Red (184), Blue (328), Green (504)

Fig. 9. Effect of dimple number.



[Black: 0, smooth ball], [Brown: 150], [Red: 300], [Blue: 400]

Fig. 10. Number of dimples *vs.* flight distance.

D. Total Drag Coefficient

The value of C_{D_size}, calculated based on the size of the dimple, the value of C_{D_depth}, calculated based on its depth, and the value of C_{D_num}, calculated based on its number, are summed as in Equation (11). We can simulate the changes of flight distance caused by the effects of different dimple characteristics through our dimple model.

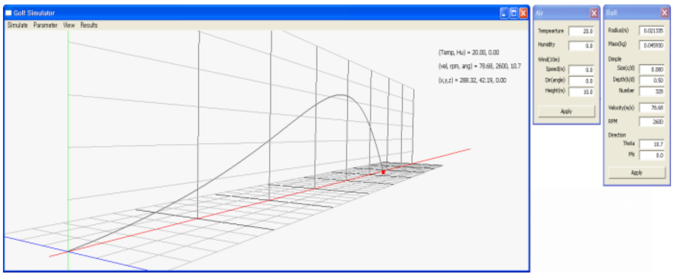
$$C_D = C_{D\_num} + C_{D\_size} + C_{D\_depth} \tag{11}$$

VI. RESULTS

Fig. 11 shows our flight trajectory simulation system. User can input temperature, humidity, and altitude for calculation of air density, and dimple characteristics (dimple size and depth ratio, dimple number). When initial velocity and angular velocity of a ball is given, our system calculates the trajectory of a ball using 4th order Runge-Kutta method.

Table II shows a comparison between measured and simulated distances. The average error is about 3.4%. As can be seen, the simulated distances are quite similar to real ones under various conditions.

Reflecting the changes in air and dimple characteristics, the flight simulation model suggested in this paper can provide varying effects for players, increasing the reality of a screen golf game by calculating the drag and lift coefficients in an exact manner.



(Temp, Hu) = 20.00, 0.00
(vel, rpm, ang) = 78.68, 2600, 10.7
(x,y,z) = 288.32, 42.19, 0.00

Tempearture 20.0
Humidity 0.0
Wind(10m)
Speed(m) 0.0
Dir(angle) 0.0
Height(m) 10.0

Radius(m) 0.021335
Mass(kg) 0.045930
Dimple
Size(c/d) 0.080
Depth(k/d) 0.50
Number 328
Velocity(m/s) 78.68
RPM 2600
Direction
Theta 10.7
Phi 0.0

Fig. 11. Simulator Interface (perspective view).

TABLE II: COMPARISON BETWEEN SIMULATED AND MEASURED DISTANCE

	V(m/s)	Angle (deg.)	Spin (rpm)	Real Carry (m)	Virtual Carry (m)	Error (%)
1	71.97	8.5	2,390	274.59	258.47	6.2
2	75.10	11.7	2,673	268.38	277.11	3.2
3	77.78	11.5	2,400	292.24	283.34	3.1
4	78.68	10.7	2600	282.46	288.66	2.2
5	79.57	13.0	2,200	281.64	287.80	2.2

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