Art of Arrangement Based on Golden Ratio and Line Segment

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***Abstract*—This research aims to explore the mathematical rules in pattern creation for the artistic work required by the designers. The proposed methodology utilized the well known golden ratio and further extended to two rules: rule based on golden rectangle and rule based on Fibonacci rectangle. The method also proposed the spiral made from line segment rule. By considering a floral outline as the 'seed' of the pattern and by applying the rules, the pattern can then be generated. This study portrays a potential direction in exploring the pattern creation based on the geometrical properties.**

***Index Terms*—Golden ratio, pattern creation, spiral.**

I. Introduction

In designing the artworks, the designers always utilize the graphical software to assist their works in creating the patterns. However, the existing graphical software tools have limited geometrical pattern plug-ins, hence this motivated the cooperation of the *researchers* from different discipline to work on developing more geometrical rules for pattern creation.

This project aims to develop various mathematical rules to create pattern, *preferably* pattern related to botanical objects. The outcome of this project can be applied to software development that will assist designers to create, explore or experiment various design concepts at various stages of their design processes. Living a life surrounded by predictable cycles either of regular patterns (repetitive and normal), or irregular patterns, in different shapes and colours – may be monotonous and boring. This project will break the monotony as observations will be made, reflected on, and related to real-life patterns found in nature. However, it is worth noting that all life patterns should be better appreciated with fewer complaints.

"Pattern" can be defined as "arrangement with regularity, order, repetition, and scale". Exploring and seeing patterns in nature is an *exciting* visual experience. The editors of Time-Life Book [1] revealed that natural resources should be discovered as distinct visual aspects of everyday object for creative opportunities. Computer can be used for designing, transforming and colouring patterns, allowing experimentation of various pattern designs. Among the popular tools normally used for designing patterns are Adobe's graphics software, Illustrator and Photoshop (www.adobe.com) and its plug-in software from Artlandia (www.artlandia.com), and Grafffix (www.graphicxtras.com). However, the software merely provides a computational platform in assisting the designing process. To design nature-based patterns either by hand sketching or even the software is not an easy process. Pattern creation through extracting properties have been done by some researchers [2] and [3] through various techniques. A representative mathematical model known as Reaction-Diffusion (RD) system has been used to explain the patterning phenomena. This model has been proven to be able to explain the specific characteristics of animal skin patterns [4] and [5]. In [6], it has presented a method built upon existing physically-based methods to generate surface crack patterns that appear in materials such as mud, ceramic glaze and glass. A research was conducted by [3] to simulate branch properties of a plant through computer simulation programs. By using simple branching rules, simulations similar to actual trees were constructed.

Many natural patterns share a similar mechanism of formation called "self-organization" which is characterized by simple "rules" that depend solely on local interactions among the sub-*units* of the system [2]. Cellular automata have been used to simulate complex self-organization patterns based on simple rules [7]. In terms of botanical value, patterns form based on self–organization is much appreciated. In their research on botanical computing, the authors [8] have listed their observations on plants: 1) plants are ubiquitous; 2) plants are informative; 3) plants are diverse; and, 4) plants are reactive. These characteristics provide potential research problems in exploring new inventions on new types of bio-interactive arts or design [8].

In this project, we are proposing some mathematical rules to designers where the pattern can be created.

II. Methodology

The proposed methodology is to develop some mathematical rules in creating the patterns. The rules developed are based on the well known golden ratio, as well as the common spiral pattern. By combining these rules with the 'seed', in which the 'seed' would act as the fundamental style, a pattern could then be generated.

Golden *ratio*, denoted as φ is an irrational constant given by

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339887\ldots,$$

with the digits go on forever without repeating [9], [10]. In the proposed method, we further extend the golden ratio to

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golden rectangle and Fibonacci rectangle.

A. Rules Based on Golden Rectangle

Golden rectangle is a rectangle in which the ratio of the length and width is in golden ratio. The pattern could be generated by recursively generating multiple golden rectangles by following the steps as described bellow:

Step 0: Given a rectangle with length a_0 and width a_1, where

$$\frac{a_0}{a_1} = \varphi .$$

Step 1: By cutting a square from the rectangle, we have one square and the remaining rectangle would be a golden rectangle. Observe that the side-length of the square is a_1 and the size of the remaining rectangle is $a_1 \times a_2$. Thus, we have

$$\frac{a_1}{a_2} = \varphi ,$$

$$a_1 + a_2 = a_0 .$$

Step 2: Repeat step 2 to the $a_1 \times a_2$ rectangle would produce a smaller square with side-length a_2, where $b_1 = a_1$ and a rectangle of size $a_2 \times a_3$ which satisfy

$$\frac{a_2}{a_3} = \varphi ,$$

$$a_2 + a_3 = a_1 .$$

Step 3: Keep cutting the remaining rectangle into a smaller square and a smaller rectangle. For each cut, the following rules have to be observed:

$$\frac{a_i}{a_{i+1}} = \varphi ,$$

$$a_i + a_{i+1} = a_{i-1} .$$

By repeating step 3 for *n* iteration, we could obtain the pattern of *embedded* golden rectangle in larger golden rectangle iteratively. Fig. 1 depicts a sample of the golden rectangle with *n* = 5.

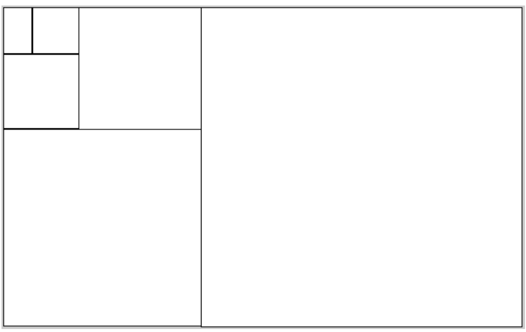

Fig. 1. Golden rectangle with *n* = 5.

B. Rules Based on Fibonacci Rectangle

Fibonacci sequence [10], [11] is a sequence where each term is the *sum* of the two previous terms, and is given by 0, 1, 1, 2, 3, 5, 8, 13, ….

If the sequence is continue further, the ratio of a term to the next term would get closer and closer to the golden ratio. By utilizing the Fibonacci sequence, Fibonacci rectangles could be generated with the *procedure* is just opposite as the one described above in generating golden rectangle. Details of generating Fibonacci rectangles are as follows:

Step 1: Draw two squares of length $a_0 = 1$ side by side.

Step 2: Add a square of length $a_1 = a_0 + a_0$ with its side must coincide with the previous two squares.

Step 3: Add another square of length $a_2 = a_1 + a_0$ with its side must coincide with the previous two squares, with length a_1 and a_0, respectively.

Step 4: Repeat adding the square of length $a_{i+1} = a_i + a_{i-1}$ and for each addition of new square, one of its sides must coincide with the previous two squares with length a_i and a_{i-1}, respectively.

By *continuing* step 4 for *n* iteration, eventually the whole constructed rectangle would be closer and closer to the golden rectangle. Again, we could obtain the pattern of embedded rectangle in a larger rectangle iteratively, though the rectangle size might not follow the golden ratio if *n* is not big enough. See Fig. 2 for a sample of Fibonacci rectangle with *n* = 5.

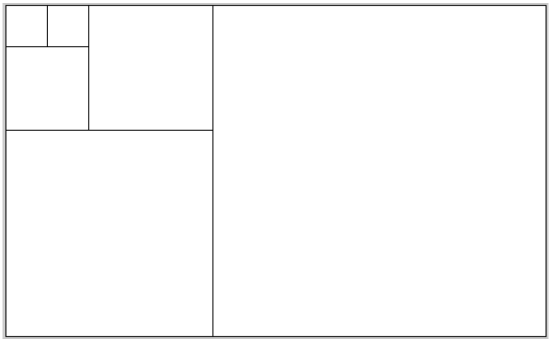

Fig. 2. Fibonacci Rectangle with *n* = 5.

C. Spiral Made of Line Segment Rule

Instead of using the golden ratio, we may also generate the line *patterns* using other simple mathematical rules. The proposed rule here is to generate spiral pattern made of line segment. The details procedure as follows:

Step 1: Draw one vertical line with length l_0. At the upper end of the line, start from the angle of π draw another line with length l_0.

Step 2: At another end of the previous line, start from the angle of $\frac{2\pi}{3}$ draw one line with length $2l_0$. At the end of this line, draw a new line with length $2l_0$ at the angle of 0.

Step 3: At another end of the pervious line, at the angle of $\frac{\pi}{2}$, draw a new line with length $3l_0$. At the end of this line, draw a new line with length $3l_0$ at the angle of π.

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Step *i*: Repeating step 2. Each addition of a new line following the order of the angle at $\frac{2\pi}{3}$ and 0, with the length of the new line is il_0.

Step i+1: Repeating step 3. Each addition of a new line following the order of the angle at $\frac{\pi}{2}$ and π, with the length of the new line is $(i+1)l_0$.

By repeating the iteration for n time, we may generate a spiral pattern *with* line segment with n = 10 (see Fig. 3).

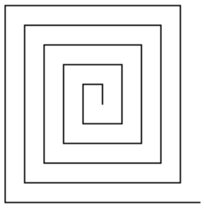
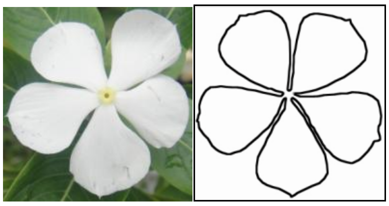


Fig. 3. Spiral made of line segment with n = 10.

By using the above three rules, the 'seed' which carries the *fundamental* pattern information could be generated, and we would then have a pattern created based on the mathematical rules.

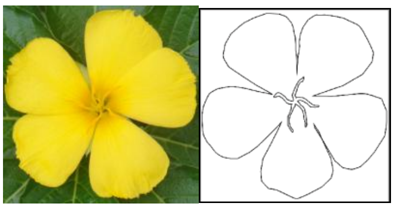
III. Results

The floral pattern has been chosen as the 'seed' in the creation of pattern following the proposed rules. Two types of flowers have been chosen, namely '*Catharanthus roseus*' and '*Turnera ulmifolia*'. Fig. 4 and Fig. 5 depict the photo and the main outlines traced to act as the 'seed' for both flowers.



(a) (b)

Fig. 4. (a) Photo of '*Catharanthus roseus*' flower. (b) Outline of '*Catharanthus roseus*' traced from photo.



(a) (b)

Fig. 5. (a) Photo of '*Turnera ulmifolia*' flower. (b) Outline of '*Turnera ulmifolia*' traced from photo.

By applying the 'seed' to the proposed three rules, the floral pattern can be generated. Fig. 6 and Fig. 7 show the *pattern* generated utilizing the rules based on golden rectangle and Fibonacci rectangles, respectively, for both '*Catharanthus roseus*' flower and '*Turnera ulmifolia*' flowers, with n = 3.

Since the iteration used is less, we can see from the patterns that there is a small variation. The pattern generated from the rules *based* on Fibonacci rectangle gives the floral all in the size of a square. Whereas, the pattern generated from the rules based on golden rectangle gives a small golden rectangle at the upper end corner.

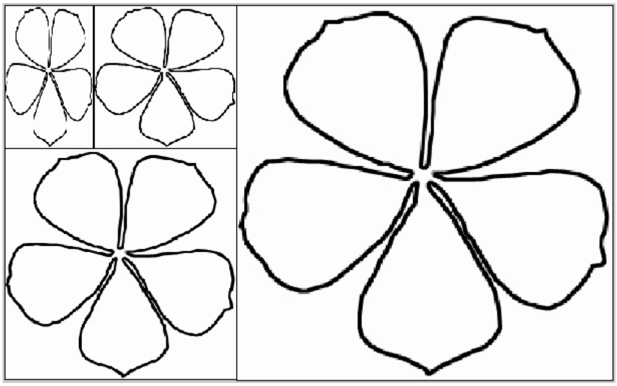


Fig. 6. (a) Floral pattern ('*Catharanthus roseus*') generated using rules based on golden rectangle with n = 3.

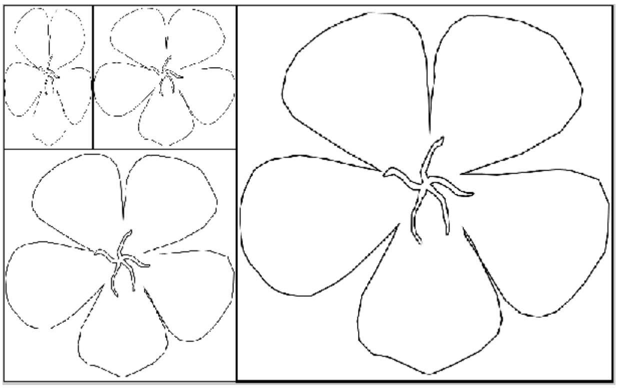


Fig. 6. (b) Floral pattern ('*Turnera ulmifolia*') generated using rules based on golden rectangle for '*Turnera ulmifolia* ' with n = 3.

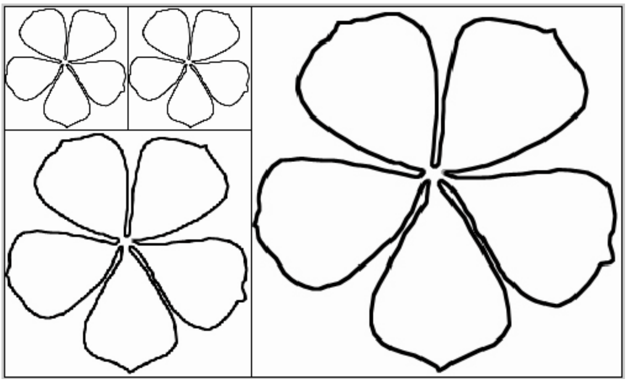


Fig. 7. (a) Floral pattern ('*Catharanthus roseus*') generated using rules based on Fibonacci rectangle with n = 3.

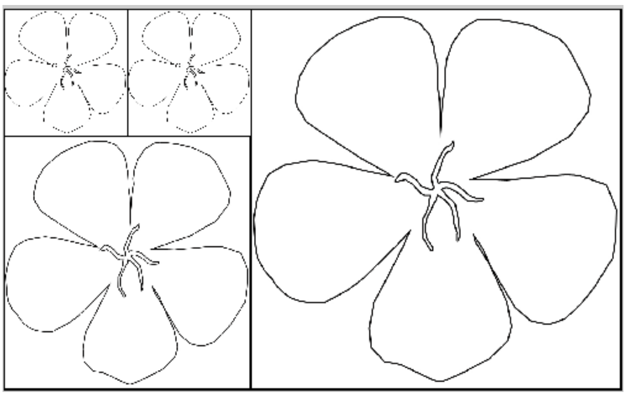


Fig. 7. (b) Floral pattern ('*Turnera ulmifolia*') generated using rules based on Fibonacci rectangle with n = 3.

If the iteration n used in rules based on golden rectangle or rules based on Fibonacci rectangle is increased, basically there is not much variation we can see from the patterns generated from both rules since the size of the golden rectangle is too small to be differentiate significantly from the square. See Figure 8 for the pattern generated utilizing the rules based on golden rectangle up to generation of 9 squares and Figure 9 gives the pattern from the rules based on Fibonacci rectangle with n = 10, for both '*Catharanthus roseus*' flower and '*Turnera ulmifolia*' flower.

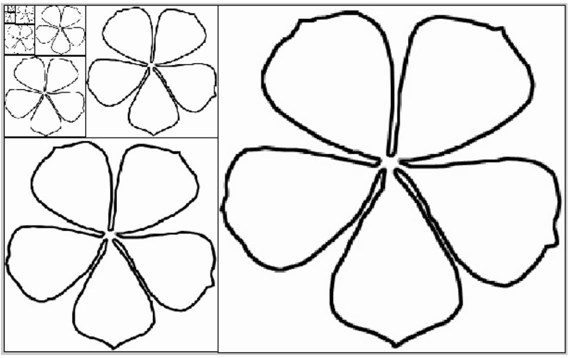


Fig. 8. (a) Floral Pattern ('*Catharanthus roseus*') generated using rules based on golden rectangle with n = 10.

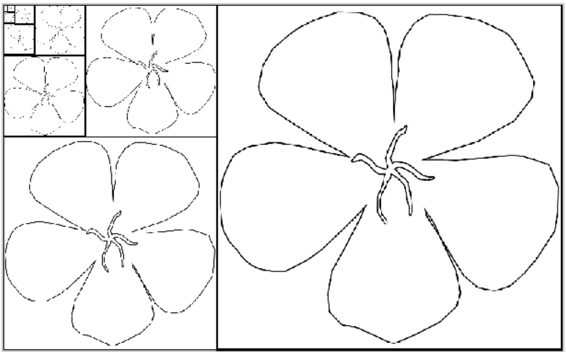


Fig. 8. (b) Floral Pattern ('*Turnera ulmifolia*') generated using rules based on golden triangle wit n = 10.

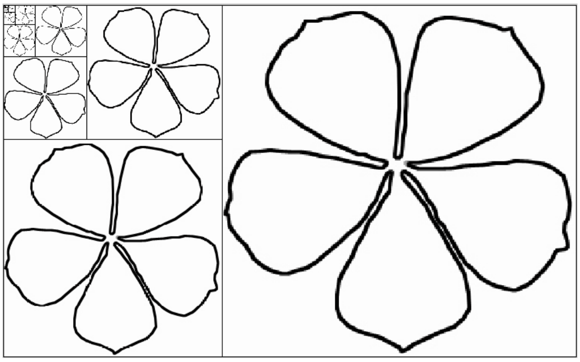


Fig. 9. (a) Floral Pattern ('*Catharanthus roseus*') generated based on rules based on Fibonacci rectangle with n = 10.

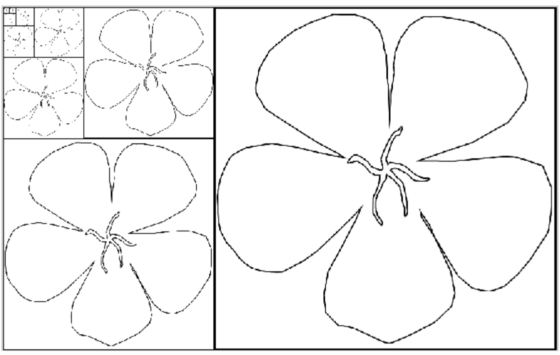


Fig. 9. (b) Floral Pattern ('*Turnera ulmifolia*') generated using rules based on Fibonacci rectangle wit n = 10.

Applying the 'seed' to the spiral made of line segment rule, the pattern *obtained* for both '*Catharanthus roseus*' and '*Turnera ulmifolia*' is as shown in Fig. 10.

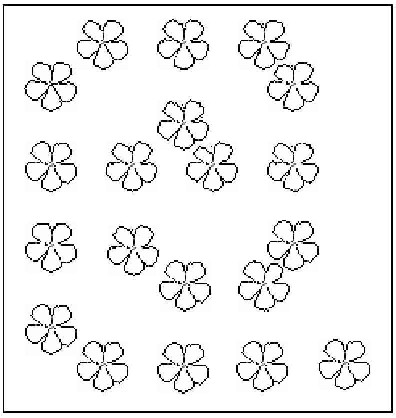


Fig. 10. (a) Floral Pattern ('*Catharanthus roseus*') generated using sprial made of line segment rule.

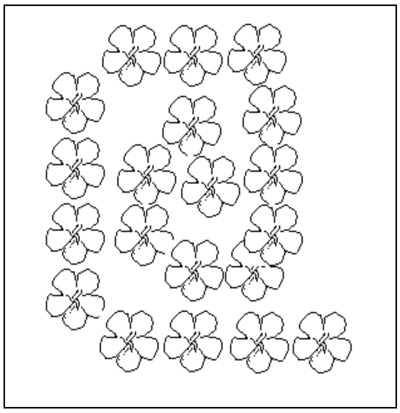


Fig. 10. (b) Floral Pattern ('*Turnera ulmifolia*') generated using sprial made of line segment rule.

IV. Conclusion and Suggestion

In this study, the ideas of mathematical properties are explored in pattern creation, and hence provide more flexibility for the designer in their art works. This research has brought the people from different disciplines to work together by *combining* the mathematical systematical approach with the artistic approach of multimedia creation field. The study is still in fundamental stage and can still be further enhanced. The future direction of this research is to extract the geometrical properties from the specific type of objects, says the tropical botanical objects, and generalizes the properties to mathematical rules for pattern creation.

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