

# Design of Optimal PID Controller for Inverted Pendulum Using Genetic Algorithm

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**Abstract**—one of the most important problems today is robotics and its control, due to the vast Application of inverted pendulum in robots. In this paper, we have tried to optimally PID Controller inverted pendulum using Genetic Algorithm by nonlinear equations. The results of this simulation have been mentioned in the conclusion. It seems that the results be acceptable results.

**Index Terms**-Nonlinear, optimal, classical PID controller, genetic algorithm.

## I. INTRODUCTION

There are variety methods for DC motors control that are presented since now. The presented methods for DC motors control are divided generally in three groups. Classic methods such as PID, PI controllers [1, 2]. Modern methods (adaptation-optimum) [3, 4, 5]. Artificial methods such as neural networks and fuzzy [6, 7].theory are the presented methods for DC motors speed control.

The design method in linear control comprise based on main application the wide span ' of frequency, linear controller has a weak application, because it can't compensate the nonlinear system effect completely.

## II. MODELING AN INVERTED PENDULUM

The cart with an inverted pendulum, shown below, is "bumped" with an impulse force, F. Determine the dynamic equations of motion for the system, and linearize about the pendulum's angle,  $\theta = 0$  (in other words, assume that pendulum does not move more than a few degrees away from the vertical, chosen to be at an angle of 0). Find a controller to satisfy all of the design requirements given below.

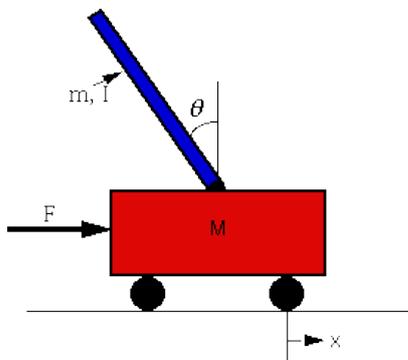


Fig. 1. The structure of an inverted pendulum.

For this example, let's assume that

TABLE I: PHYSICAL PARAMETERS OF INVERTED PENDULUM

M	mass of the cart	0.5 kg
m	mass of the pendulum	0.2 kg
b	friction of the cart	0.1 N/m/sec
l	length to pendulum center of mass	0.3 m
I	inertia of the pendulum	0.006 kg*m^2
F	force applied to the cart	
x	cart position coordinate	
theta	pendulum angle from vertical	

This system is tricky to model in Simulink because of the physical constraint (the pin joint) between the cart and pendulum which reduces the degrees of freedom in the system. Both the cart and the pendulum have one degree of freedom (X and theta, respectively). We will then model Newton's equation for these two degrees of freedom.

$$\frac{d^2x}{dt^2} = \frac{1}{M} \sum_{cart} F_x = \frac{1}{M} \left( F - N - b \frac{dx}{dt} \right) \quad (1)$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{I} \sum_{pend} \tau = \frac{1}{I} (NL \cos(\theta) + PL \sin(\theta)) \quad (2)$$

It is necessary, however, to include the interaction forces N and P between the cart and the pendulum in order to model the dynamics. The inclusion of these forces requires modeling the x and y dynamics of the pendulum in addition to its theta dynamics. Generally, we would like to exploit the modeling power of Simulink and let the simulation take care of the algebra. Therefore, we will model the additional x and y equations for the pendulum.

$$m \frac{d^2x_p}{dt^2} = \sum_{pend} F_x = N \quad (3)$$

$$\Rightarrow N = m \frac{d^2x_p}{dt^2} \quad (4)$$

$$m \frac{d^2y_p}{dt^2} = P - mg \quad (5)$$

$$\Rightarrow P = m \left( \frac{d^2y_p}{dt^2} + g \right) \quad (6)$$

However,  $x_p$  and  $y_p$  are exact functions of theta. Therefore, we can represent their derivatives in terms of the derivatives of theta.

$$x_p = x - L \sin(\theta) \quad (7)$$

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$$\frac{dx_p}{dt} = \frac{dx}{dt} - L \cos(\theta) \frac{d^2\theta}{dt^2} \quad (8)$$

$$\frac{d^2x_p}{dt^2} = \frac{d^2x}{dt^2} + L \sin(\theta) \left(\frac{d\theta}{dt}\right) - L \cos(\theta) \frac{d^2\theta}{dt^2} \quad (9)$$

$$y_p = L \cos(\theta) \quad (10)$$

$$\frac{dy_p}{dt} = -L \sin(\theta) \frac{d\theta}{dt} \quad (11)$$

$$\frac{d^2y_p}{dt^2} = -L \cos(\theta) \left(\frac{d\theta}{dt}\right)^2 - L \sin(\theta) \frac{d^2\theta}{dt^2} \quad (12)$$

These expressions can then be substituted into the expressions for N and P. Rather than continuing with algebra here, we will simply represent these equations in Simulink.

Simulink can work directly with nonlinear equations, so it is unnecessary to linearize these equations.

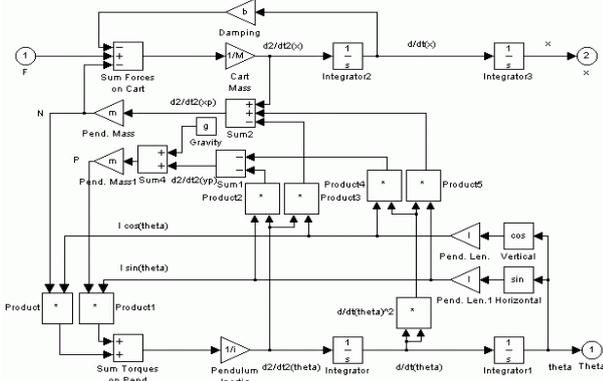


Fig. 2. The block diagram of an inverted pendulum.

### III. GENETIC ALGORITHM

In this algorithm, first of all, we create some random populations. Every individual (gene) In GA is considered in the form of binary strings then, fitness for every individual is chosen with regard to its fitness.

For creating the next generation, three stages is the selection phase, which consists of different phases, including ranking, proportional and... The second phase is the combination phase. In this phase, the two parents are

combined with pc possibility and the next generation comes in to being.

By considering that during the past phases of gene it may cause noise, in fact, this phase is a Random noise which causes a small pc possibility for every bit.

For GA, in every problem, a fitness function must be defined. F functions can be described as follows:

$$F = \text{OverShoot} + \text{Ess} \quad (13)$$

$$F = A * \text{OverShoot} + B * \text{Ess} \quad (14)$$

$$F = e^{A * \text{OverShoot} + B * \text{Ess}} \quad (15)$$

In this problem, the aim is to minimize every function of F. As GA Has the ability to be maximized, hence, fitness function is defined as below.

$$\text{Fitness} = K - F \quad (16)$$

$$\text{Fitness} = \frac{1}{F} \quad (17)$$

If the fitness function is selected from an equation (16) constant parameter k must be regulated in a way that causes no harm to the problem. If k is a small number, fitness will be negative and for the capital k, the fitness of all the individuals in the society will be approximated. In this paper, some equations have been used.

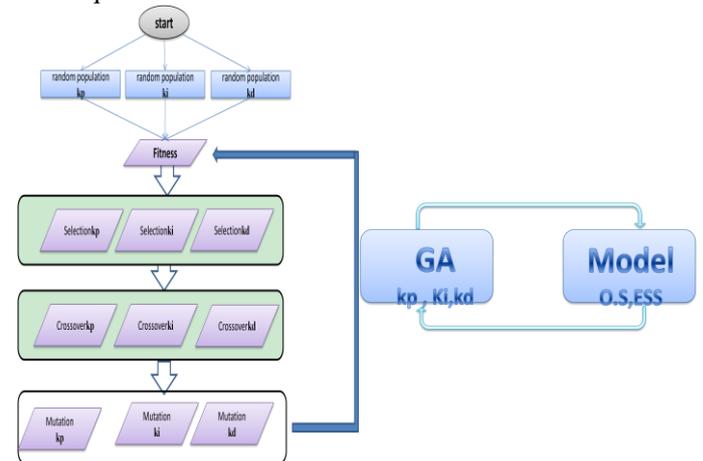


Fig. 3. Chart of genetic algorithm.

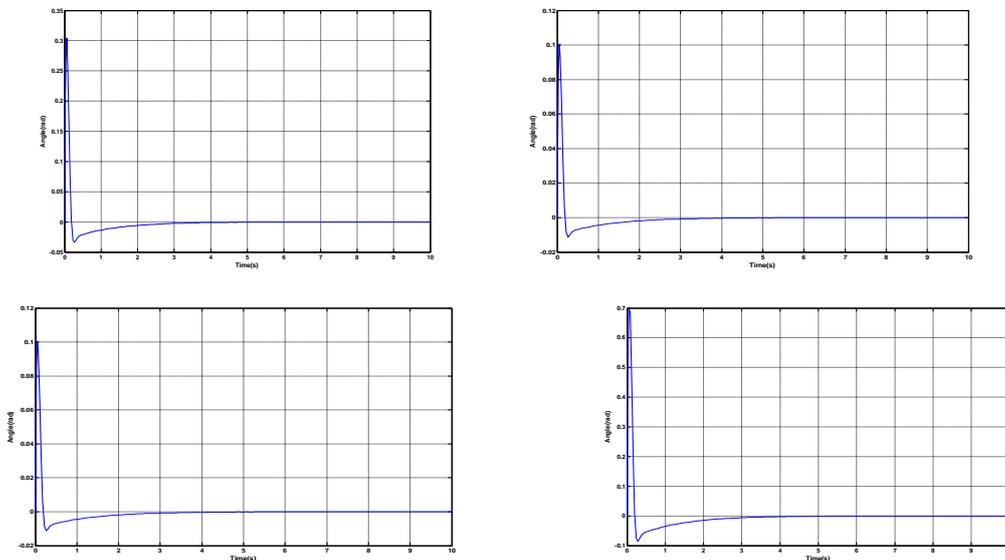


Fig. 4. results of system by Populations size=50 ; binary strings =16bit

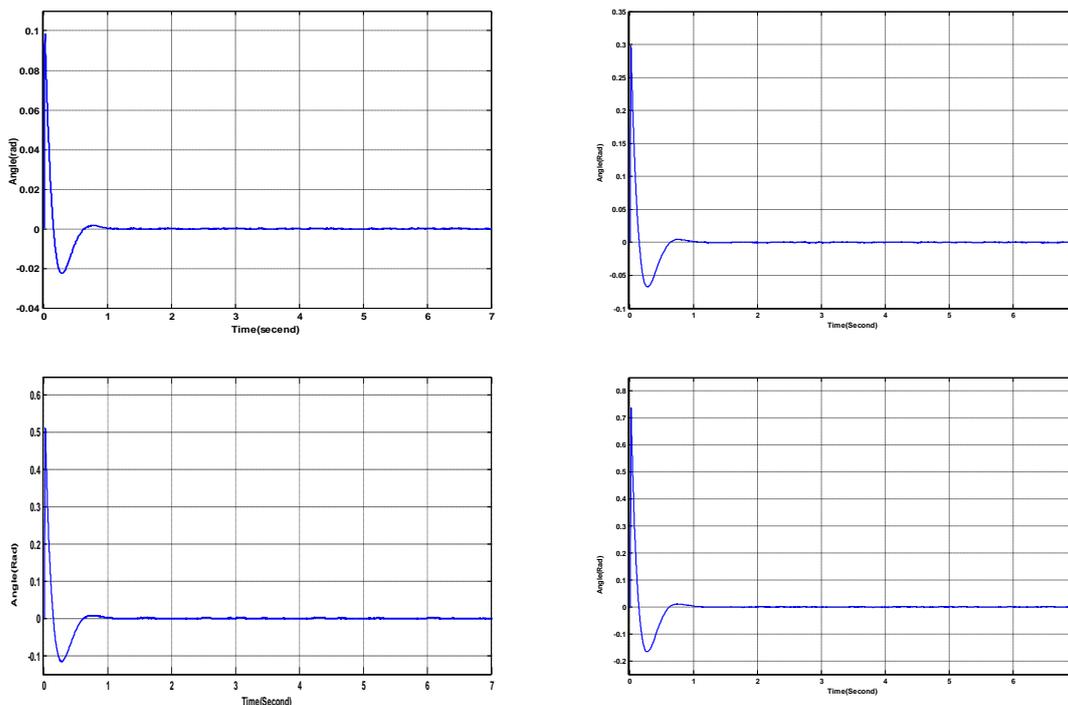


Fig. 5. Results of system by populations size=25; binary strings =8bit.

TABLE II: TUNING PARAMETER

Parameter	Value
Lower bound [Kp Ki Kd]	[0 0 0]
Upper bound [Kp Ki Kd]	[100 100 100]
Stopping criteria (Iterations)	100
Population Size	50 and 25
Crossover Fraction	0.8
Mutation Fraction	0.01
binary strings	16bit and 8 bit

IV. CONCLUSION

Parameters adjustment at different problems takes more time up by hard mathematical calculating. At this paper was tried one simple application from Genetic algorithm considered by controlengineeringproblem. We can find the optimal answer with Genetic algorithm .This answer should be careful and simple rarely acceptable.

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