International Journal of Innovation, Management and Technology, Vol. 1, No. 1, April 2010
ISSN: 2010-0248

A Constrained Inventory Level Optimal Control on Supply Chain Management System

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***Abstract*—Supply chain management system is a network of facilities and distribution entities: suppliers, manufacturers, distributors, retailers. The control system aims at operating the supply chain at the optimal point despite the influence of demand changes. In fact, the main objectives of the control strategy for the supply chain network can be summarized as follows: maximize customer satisfaction, and minimize supply chain operating costs. In this work, based on the fact that past and present control actions affect the future response of the system, a receding time horizon optimal control or model predictive control (MPC) is efficient. Also since a centralized control scheme may not suitable or even possible for technical or commercial reasons, it is useful to have decentralized control schemes. In this method, each node completely by a constrained decentralized MPC optimizes locally for its own policy. At each time period, the first decentralized predictive control action in the calculated sequence is implemented until MPC process complete. So As locally constrained predictive controllers applying to a supply chain management system consist of two plant, three warehouses, four distribution centers and four retailers.**

***Index Terms*—Supply chain management, Demand, Optimal control, Predictive control.**

I. INTRODUCTION

The network of suppliers, manufacturers, distributors and retailers constitutes a supply chain management system. Between interconnected entities, there are two types of process flows: information flows, e.g., an order requesting goods, and material flows, i.e., the actual shipment of goods (Figure 1). Key elements to an efficient supply chain are accurate pinpointing of process flows and timing of supply needs at each entity, both of which enable entities to request items as they are needed, thereby reducing safety stock levels to free space and capital. The operational planning and direct control of the network can in principle be addressed by a variety of methods, including deterministic analytical models and stochastic analytical models, and simulation models, coupled with the desired optimization objectives and network performance measures [1].

Manuscript received Dec 15, 2009.

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The significance of the basic idea implicit in the MPC has been recognized a long time ago in the operations management literature as a tractable scheme for solving stochastic multi period optimization problems, such as production planning and supply chain management, under the term receding horizon [2]. In a recent paper [3], a model predictive control strategy was employed for the optimization of production/distribution systems, including a simplified scheduling model for the manufacturing function. The suggested control strategy considers only deterministic type of demand, which reduces the need for an inventory control mechanism [4,5].

For the purposes of our study and the time scales of interest, a discrete time difference model is developed [6]. The model is applicable to multi echelon supply chain networks of arbitrary structure. To treat process uncertainty within the deterministic supply chain network model, a model predictive control approach is suggested [7,8].

Typically, MPC is implemented in a centralized fashion [7]. The complete system is modeled, and all the control inputs are computed in one optimization problem. In large scale applications, such as power systems, water distribution systems, traffic systems, manufacturing systems, and economic systems, such a centralized control scheme may not suitable or even possible for technical or commercial reasons [8,9], it is useful to have distributed or decentralized control schemes, where local control inputs are computed using local measurements and reduced order models of the local dynamics. The algorithm uses a receding horizon, to allow the incorporation of past and present control actions to future predictions [10,11]. As well as, further decentralized MPC advantages are less computational complication and lower error risk [12,13].

Since supply chains can be operated sequentially, locally consecutive predictive controllers hierarchically applying to a supply chain management system consist of two plant, three warehouses, four distribution centers and four retailers.

II. DYNAMIC MODEL

In this work, a discrete time difference model is developed[4]. The model is applicable to multi echelon supply chain networks of arbitrary structure, that *DP* denote the set of desired products in the supply Chain and these can be manufactured at plants, ***P***, by utilizing various resources, *RS*. The manufacturing function considers independent production lines for the distributed products. The products are subsequently transported to and stored at warehouses, ***W***.

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Products from warehouses are transported upon customer demand, either to distribution centers, *D*, or directly to retailers, *R*. Retailers receive time varying orders from different customers for different products. Satisfaction of customer demand is the primary target in the supply chain management mechanism. Unsatisfied demand is recorded as backorders for the next time period. A discrete time difference model is used for description of the supply chain

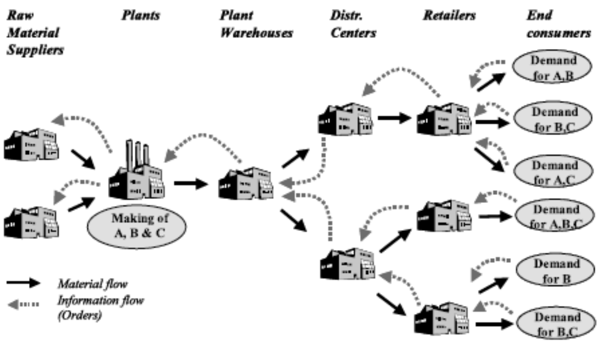

Figure 1. Schematic of a multi echelon/multi product (A, B, C) supply chain network with process flows.

Network dynamics. It is assumed that decisions are taken within equally spaced time periods (e.g. hours, days, or weeks). The duration of the base time period depends on the dynamic characteristics of the network. As a result, dynamics of higher frequency than that of the selected time scale are considered negligible and completely attenuated by the network[4,14].

Plants *P*, warehouses *W*, distribution centers *D*, and retailers *R* constitute the nodes of the system. For each node, *k*, there is a set of upstream nodes and a set of downstream nodes, indexed by (k', k''). Upstream nodes can supply node *k* and downstream nodes can be supplied by *k*. All valid (k', k) and/or (k, k'') pairs constitute permissible routes within the network. All variables in the supply chain network (e.g. inventory, transportation loads) valid for bulk commodities and products. For unit products, continuous variables can still be utilized, with the addition of a post processing rounding step to identify neighbouring integer solutions. This approach, though clearly not formally optimal, may be necessary to retain computational tractability in systems of industrial relevance.

A product balance around any network node involves the inventory level in the node at time instances *t* and *t* − 1, as well as the total inflow of products from upstream nodes and total outflow to downstream nodes. The following balance equation is valid for nodes that are either warehouses or distribution centers:

$$y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k',k}) - \sum_{k''} x_{i,k,k''}(t), \qquad (1)$$
$$\forall k \in \{W, D\}, \qquad t \in T, \qquad i \in DP$$

where $y_{i,k}$ is the inventory of product *i* stored in node *k*; $x_{i,k',k}$ denotes the amount of the *i*-th product transported through route (k', k); $L_{k',k}$ denotes the transportation lag (delay time) for route (k', k), i.e. the required time periods for the transfer of material from the supplying node to the current node. The transportation lag is assumed to be an integer multiple of the base time period.

For retailer nodes, the inventory balance is slightly modified to account for the actual delivery of the *i*-th product attained, denoted by $d_{i,k}(t)$.

$$y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k',k}) - d_{i,k}(t), \qquad (2)$$
$$\forall k \in \{R\}, \qquad t \in T, \qquad i \in DP.$$

The amount of unsatisfied demand is recorded as backorders for each product and time period. Hence, the balance equation for back orders takes the following form:

$$BO_{i,k}(t) = BO_{i,k}(t-1) + R_{i,k}(t) - d_{i,k}(t) - LO_{i,k}(t), \qquad (3)$$
$$\forall k \in \{R\}, \qquad t \in T, \qquad i \in DP.$$

where $R_{i,k}$ denotes the demand for the *i*-th product at the *k*-th retailer node and time period *t*. $LO_{i,k}$ denotes the amount of cancelled back orders (lost orders) because the network failed to satisfy them within a reasonable time limit. Lost orders are usually expressed as a percentage of unsatisfied demand at time *t*. Note that the model does not require a separate balance for customer orders at nodes other than the final retailer nodes [4,15].

III. Control

MPC originated in the late seventies and has developed considerably since then. The term model predictive control does not designate a specific control strategy but rather an ample range of control methods which make explicit use of a model of the process to obtain the control signal by minimizing an objective function. The ideas, appearing in greater or lesser degree in the predictive control family, are basically the explicit use of a model to predict the process output at future time instants (horizon), the calculation of a control sequence minimizing an objective function and the use of a receding strategy, so that at each instant the horizon is displaced towards the future, which involves the application of the first control signal of the sequence calculated at each step. The success of MPC is due to the fact that it is perhaps the most general way of posing the control problem in the time domain. The use a finite horizon strategy allows the explicit handling of process and operational constraints by the MPC. The control system aims at operating the supply chain at the optimal point despite the influence of demand changes [12,13]. The control system is required to possess built in capabilities to recognize the optimal operating policy through meaningful and descriptive cost performance indicators and mechanisms to successfully alleviate the detrimental effects of demand uncertainty and variability. The main objectives of the control strategy for the supply chain network can be summarized as follows: (i) maximize customer satisfaction, and (ii) minimize supply chain operating costs.

The first target can be attained by the minimization of back orders (i.e. unsatisfied demand) over a time period because unsatisfied demand would have a strong impact on company reputation and subsequently on future demand and total revenues. The second goal can be achieved by the minimization of the operating costs that include transportation and inventory costs that can be further divided into storage costs and inventory assets in the supply chain network. Based on the fact that past and present control actions affect the future response of the system, a receding time horizon is selected. Over the specified time horizon the future behavior of the supply chain is predicted using the described difference model (Eqs. (1)–(3)). In this model, the state variables are the product inventory levels at the storage nodes, y, and the back orders, BO, at the order receiving nodes. The manipulated (control or decision) variables are the product quantities transferred through the network's permissible routes, x, and the delivered amounts to customers, d. Finally, the product back orders, BO, are also matched to the output variables. The inventory target levels (e.g. inventory setpoints) are time invariant parameters. The control actions that minimize a performance index associated with the outlined control objectives are then calculated over the receding time horizon. At each time period the first control action in the calculated sequence is implemented. The effect of unmeasured demand disturbances and model mismatch is computed through comparison of the actual current demand value and the prediction from a stochastic disturbance model for the demand variability. The difference that describes the overall demand uncertainty and system variability is fed back into the model predictive control scheme at each time period facilitating the corrective action that is required.

The centralized mathematical formulation of the performance index considering simultaneously back orders, transportation and inventory costs takes the following form[4]:

$$
\begin{aligned}
J_{total} = & \\
& \sum_{t}^{t+P} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{y,i,k} (y_{i,k}(t) - y_{s,i,k}(t))^2 \right\} \\
& + \sum_{t}^{t+M} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{x,i,k',k} (x_{i,k',k}(t))^2 \right\} \\
& + \sum_{t}^{t+P} \sum_{k\in\{R\}} \sum_{i\in DP} \left\{ w_{BO,i,k} (BO_{i,k}(t))^2 \right\} \\
& + \sum_{t}^{t+M} \sum_{k\in\{W,D,R\}} \sum_{i\in DP} \left\{ w_{\Delta x,i,k',k} (x_{i,k',k}(t) - x_{i,k',k}(t-1))^2 \right\}
\end{aligned} \tag{4}
$$

The performance index, J, in compliance with the outlined control objectives consists of four quadratic terms. Two terms account for inventory and transportation costs throughout the supply chain over the specified prediction and control horizons (P , M). A term penalizes back orders for all products at all order receiving nodes (e.g. retailers) over the prediction horizon P. Also a term penalizes deviations for the decision variables (i.e. transported product quantities) from the corresponding value in the previous time period over the control horizon M. The term is equivalent to a penalty on the rate of change in the manipulated variables and can be viewed as a move suppression term for the control system. Such a policy tends to eliminate abrupt and aggressive control actions and subsequently, safeguard the network from saturation and undesired excessive variability induced by sudden demand changes. In addition, transportation activities are usually preferred to resume a somewhat constant level rather than fluctuate from one time period to another.

However, the move suppression term would definitely affect control performance leading to a more sluggish dynamic response. The weighting factors, $w_{y,i,k}$, reflect the inventory storage costs and inventory assets per unit product, $w_{x,i,k',k}$, account for the transportation cost per unit product for route (k',k) .Weights $w_{BO,i,k}$ correspond to the penalty imposed on unsatisfied demand and are estimated based on the impact service level has on the company reputation and future demand. Weights $w_{\Delta x,i,k',k}$, are associated with the penalty on the rate of change for the transferred amount of the i-th product through route (k',k). Even though, factors $w_{y,i,k}$, $w_{x,i,k',k}$ and $w_{BO,i,k}$ are cost related that can be estimated with a relatively good accuracy, factors $w_{\Delta x,i,k',k}$ are judged and selected mainly on grounds of desirable achieved performance.

The weighting factors in cost function also reflect the relative importance between the controlled (back orders and inventories) and manipulated (transported products) variables. Note that the performance index of cost function reflects the implicit assumption of a constant profit margin for each product or product family. As a result, production costs and revenues are not included in the index.

But in this paper, a consecutive decentralized formulation will used, namely centralized cost function divided to decentralized cost functions for each stage (warehouse, distribution center, retailer):

$$
\begin{aligned}
J_1 = & \\
& \sum_{t}^{t+P} \sum_{i\in DP} \left\{ w_{y,i,k} (y_{i,k}(t))^2 \right\} \\
& + \sum_{t}^{t+M} \sum_{i\in DP} \left\{ w_{x,i,k',k} (x_{i,k',k}(t))^2 \right\} \\
& + \sum_{t}^{t+M} \sum_{i\in DP} \left\{ w_{\Delta x,i,k',k} (x_{i,k',k}(t) - x_{i,k',k}(t-1))^2 \right\}, \quad k\in W
\end{aligned} \tag{5}
$$

$$J_2 = \sum_{t}^{t+P}\sum_{i\in DP}\left\{w_{y,i,k}(y_{i,k}(t))^2\right\} + \sum_{t}^{t+M}\sum_{i\in DP}\left\{w_{x,i,k',k}(x_{i,k',k}(t))^2\right\} + \sum_{t}^{t+M}\sum_{i\in DP}\left\{w_{\Delta x,i,k',k}(x_{i,k',k}(t)-x_{i,k',k}(t-1))^2\right\},\quad k\in D \tag{6}$$

$$J_3 = \sum_{t}^{t+P}\sum_{i\in DP}\left\{w_{y,i,k}(y_{i,k}(t))^2\right\} + \sum_{t}^{t+M}\sum_{i\in DP}\left\{w_{x,i,k',k}(x_{i,k',k}(t))^2\right\} + \sum_{t}^{t+P}\sum_{i\in DP}\left\{w_{BO,i,k}(BO_{i,k}(t))^2\right\} + \sum_{t}^{t+M}\sum_{i\in DP}\left\{w_{\Delta x,i,k',k}(x_{i,k',k}(t)-x_{i,k',k}(t-1))^2\right\},\quad k\in R. \tag{7}$$

Therefore by this implementation, as supply chains can be operated sequentially, i.e., stages update their policies in series, synchronously, each node by a decentralized MPC optimizes locally for its own policy, and communicates the most recent policy to those nodes to which it is coupled. In fact, MPC corresponding to retailers (with Eqs. (1),(5)) only will optimized locally for its own policy and then will sent its optimal inputs to upstream joint nodes to those nodes which it is coupled (distribution centers), as measurable disturbances. Also MPC corresponding to distribution centers (with Eqs. (1),(6)) only will optimized locally for its own policy and then will sent its optimal inputs to upstream joint nodes to those nodes which it is coupled (warehouse centers), as measurable disturbances. Finally, MPC corresponding to warehouses (with Eqs. (2),(3),(7)) will optimized locally for its own optimal inputs.

Each node completely by a decentralized MPC optimizes locally for its own policy. At each time period, the first decentralized predictive control action in the calculated sequence is implemented until MPC process complete. In fact, decentralized MPC corresponding to retailers will done locally for regulating inventory level in R and then will sent its predictive control optimal inputs at long prediction horizon to upstream joint nodes to those nodes which it is coupled (distribution centers), as measurable disturbances. Also MPC corresponding to distribution centers will optimized locally and then will sent its optimal inputs to upstream joint nodes to those nodes which it is coupled (warehouse centers), as measurable disturbances. Finally, MPCs of warehouses will optimized locally for its own optimal inputs. In fact decentralized MPCs, independently and sequentially, operate (Figure 2).

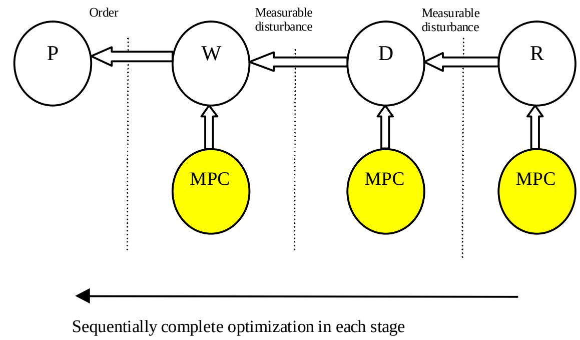

Figure 2. Procedure of hierarchically consecutive decentralized MPC

IV. Simulations

A four echelon supply chain system is used in the simulated examples. The supply chain network consists of two production nodes, three warehouse nodes, four distribution centers, and four retailer nodes (Figure 3).

All possible connections between immediately successive echelons are permitted. One product is being distributed through the network. Inventory setpoints, maximum storage capacities at every node, and transportation cost data for each supplying route are reported in Table 1.

A prediction horizon of 30 time periods and a control horizon of 15 time periods were selected and was considered $LO_{i,k}=1$ for every times. So each delay was replaced by its 4th order Pade approximation (after system model transformed to continuous time model and then returned to discrete time model).

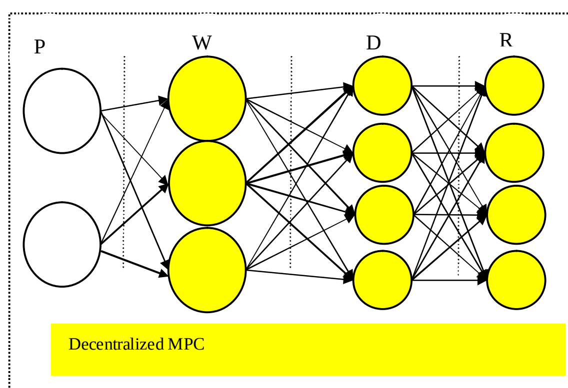

Figure 3. Schematic representation of the predictive control framework in a multiechelon supply chain

TABLE 1. Supply chain data

Echelon	W	D	R
Max inventory level	900	400	130
Inventory setpoint	400	180	65
Transportation cost(move suppression cost)	P to W 0.6	W to D 0.5	D to R 0.7
Inventory weights	1	1	1

Back-order weights	-	-	0.8
Delays	4	2	1

In this part, method of consecutive decentralized MPC that beforehand was stated, applying to large scale supply chain to constant demands equal 26.

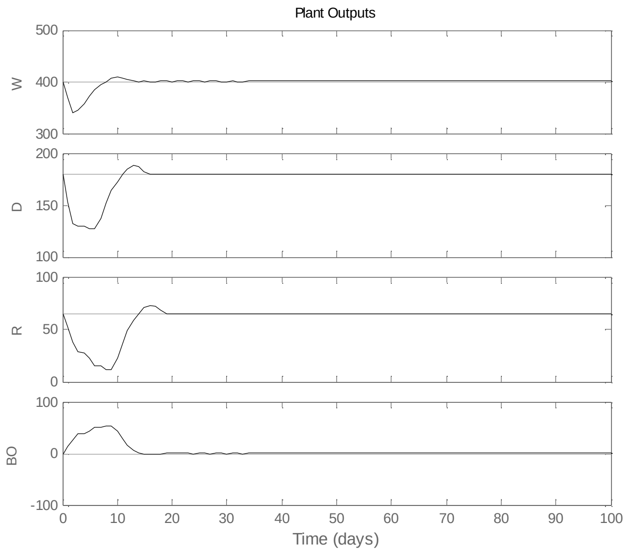

Figure 4. Discrete time dynamic response to a 26 unit constant demand for networks with different transportation delays $L = [4\ 2\ 1]$

The simulated scenarios lasted for 100 time periods. Response to constant demand is presented in figure 4 (average inventory levels in each echelon).

The move suppression term would definitely affect control performance leading to a more sluggish dynamic response. In this part, decentralized MPC method is applied to the supply chain network with pulsatory variations of customer demand that is seeing in figure 5, once with no move suppression term (Figure 6), and once with move suppression term (Figure 7).

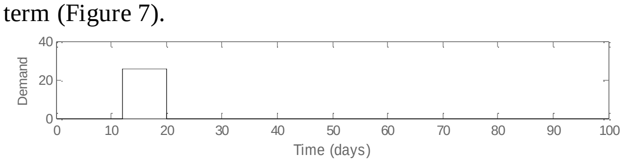

Figure 5. Pulsatory demand

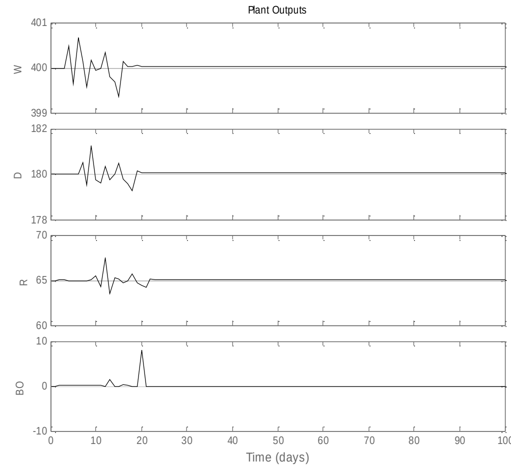

Figure 6. Inventory levels control by consecutive decentralized MPC toward discrete pulsatory demand with no move suppression effect

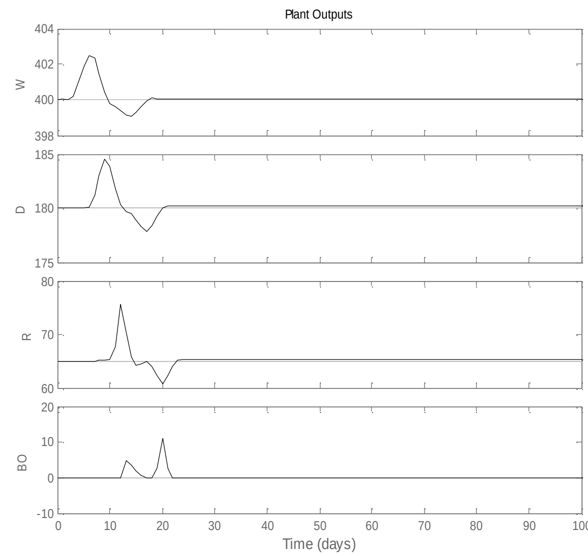

Figure 7. Inventory levels control by consecutive decentralized MPC toward discrete pulsatory demand with move suppression effect

V. Conclusion

The large majority of successful MPC applications address the case of multivariable control in the presence of constraints, motivating its extensive distribution for applications where traditional control usually comes close to its limits. The success of MPC is due to the fact that it is perhaps the most general way of posing the control problem in the time domain. The use a finite horizon strategy allows the explicit handling of process and operational constraints by the MPC. Typically, MPC is implemented in a centralized fashion. In large scale applications, such as manufacturing systems, and economic systems, such a centralized control scheme may not suitable or even possible for technical or commercial reasons, it is useful to have distributed or decentralized control schemes, where local control inputs are computed using local measurements and reduced order models of the local dynamics. As well as, further decentralized MPC advantages are less computational complication and lower error risk. So As supply chains can be operated sequentially, locally Consecutive predictive controllers applying to a supply chain management system consist of four echelon. In applied sequential decentralized MPC method, each node completely by a decentralized MPC optimized locally for its own policy hierarchically. Also a move suppression term add to cost function, that increase system robustness toward changes on demands. Through illustrative simulations with variation of demand, it is demonstrated when demand is not constant, move suppression effect decreases maximum changes of customer demands.

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