

Flexible Investment Design

Agus Ristono

Abstract— In this paper, we address the problem of designing flexible investment model where firm's product demand is subject to variability. A flexible investment model is one that maintains maximize firm's revenue despite fluctuations in the firm's product demand levels. We extend existing procedures for design of investment by explicitly capturing the stochastic nature of firm's product demand and the resulting variability in the price and (2) be determined simultaneously with the price of firm's product output as a infers function of demand scenarios. Heuristic method is presented for generating flexible investment and determining allocations of spending under various demand conditions and other assumptions.

Index Terms— flexible investment, firm's revenue, firm's product demand, firm's product output.

I. INTRODUCTION

Companies need to learn from the practices of the business funding. Many companies have had an immature approach to strategic investment management. Investment is defined by the International Monetary Fund (IMF) as the summation of gross fixed capital formation and increase (decrease) in stocks (Mehanna, 2001). Successfully integrating the client's investment policy with a thoughtful financial plan should make each asset class more productive.

The neoclassical investment model developed by Jorgenson (1963) guides the specification of the basic determinants of investment spending. In this framework, sales is the quantity variable, and the user cost of capital is the price variable (The user cost variable contains an interest rate, and is key to evaluating the effects of monetary policy on investment spending). Breitung et al (2003) proposed a new framework for studying the effects of monetary policy on business investment. Important ambiguities with the modelling of investment dynamics and interactions between real and financial decisions suggest modelling investment spending.

This research assume that firms operate in competitive markets and minimize cost; thus, price and quantity variables are exogenous. Recent research on finance constraints has demonstrated that cash flow is an important determinant of investment spending, and then Breitung et al (2003) include cash flow as financial variable. That research has also shown that financial decisions affect and are affected by real investment decisions and, hence, both investment and cash flow are endogenous. Furthermore, current cash flow shocks

affect current investment but, owing to gestation and time-to-build lags, investment does not impact cash flow contemporaneously.

Heikkinen and Pietola (2006) proposed investment model based on the dynamic optimization model, where investments are studied from the point of view of a representative dairy firm, assuming a Markov model of expectations. This research addresses the role of expectations about future payments to investment decisions in a dynamic setting. In a dynamic setting expectations of future policies are relevant for investment decisions. Heikkinen (2007) studies the effect of decoupling on optimal investment assuming disinvestment flexibility. This research is assuming decreasing income expectations. The income being determined by policy processes is subject to policy uncertainty. Greater income stability may increase investments even if accompanied by a lower subsidy level. This study examples suggest that assuming disinvestment flexibility and a higher level of investments can be achieved even with lower subsidies. Increased income stability may also diminish the dynamic cost of income uncertainty.

The limitation of previous researches on investment, such as describing at above is not robust in all market demand condition. In this paper, we address the problem of designing flexible investment where product market demand is subject to fluctuation. We define flexible investment as those that can effectively cope with market variability, where effectiveness is measured by expected investment over the various possible market scenarios. A flexible investment is, thus, one that would maintain low investment costs but maximize the firm's revenue, despite fluctuations in the market levels and fluctuations in the resulting investment flows. In our solution method, we extend existing procedures for design of flexible investment by (1) explicitly capturing the stochastic nature of market demand and the resulting investment flows between different processing, and (2) be determined simultaneously with the firm's product output as a infers function of the demand scenarios.

II. THE Q MODEL WITH INFRES FUNCTION OF THE FIRM'S OUTPUT

The model we consider is standard in the investment literature. The objective of the firm when choosing investment at time t is to maximize the present revenue of the stream of current and expected future net distributions to its existing shareholders. Assuming, for simplicity, no taxes and no debt finance, the net distribution to shareholders (i.e., dividends paid minus the revenue of new shares issued) coincides with the net revenue generated by the firm in each period. Thus the firm's objective is to maximize (Bon and Cummins, 2001):

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$$V_t = E_t \left[\sum_{s=0}^{\infty} \beta_{t+s} \pi_{t+s} \right] \quad (1)$$

where:

Π_{t+s} : net revenue generated in period $t+s$.

β_{t+s} : the discount factor used in period t to discount expected revenue in period $t+s$, with $\beta_t=1$.

$E_t[\cdot]$: an expectation conditioned on information available in period t .

We specify the net revenue function to have the form (Bon and Cummins, 2001):

$$R_t(K_t, L_t, I_t) = p_t F(K_t, L_t) - \omega_t L_t - p_t^K [I_t + G(I_t, K_t)] \quad (2)$$

where:

K_t : the stock of capital in period t ,

L_t : variable inputs used in period t ,

I_t : gross investment in period t ,

p_t : the price of the firm's output,

ω_t : prices/wage rates for the variable inputs,

p_t^K : the price of capital goods in period t .

$F(K_t, L_t)$: the production function.

$G(I_t, K_t)$: the adjustment cost function.

Hence, there are two important characteristics of real world markets: (1) dynamics and (2) competition. These issues have for a long time been neglected or treated in an inadequate way in the literature as is evidenced, for example, by that large body of research in economic, but also the marketing approach to optimal pricing seems to be that until recently neither the dynamic aspects of decision making nor the effects of actual or potential competition were satisfactorily modelled. Thus, some models, using game theory, offered promising approach to deal with competitive interactions but in many cases these models employed unreasonably simplified descriptions of the market processes. There are three major factors influence a firm's optimal pricing policy (Dockner and Jorgensen, 1988):

- 1) Diffusion effects on the demand side.
- 2) Cost experience effects.
- 3) Market structure.

So that, one approach which adopted in the investment model is the price is infers function of the demand. Since notation i as the type of firm's product output, then the Q model in eq. (1) can be modified as:

$$R_{it}(K_{it}, L_{it}, I_{it}) = p_{it}^{-1}(D_{it})F(K_{it}, L_{it}) - \omega_{it}L_{it} - p_{it}^K [I_{it} + G(I_{it}, K_{it})] \quad (3)$$

Our timing assumption is that current investment is immediately productive, and the stock of capital evolves according to:

$$K_{t+s} = (1 - \delta)K_{t+s-1} + I_{t+s} \quad (4)$$

where δ is the rate of depreciation. We also assume that current prices and the realizations of current technology shocks are known to the firm when choosing current investment. The expected revenue in equation (1) is taken over the distribution of future prices and technology shocks. Other timing conventions are certainly possible, but would not affect the substance of our analysis in the following sections.

Further assuming that adjustment costs have the symmetric, quadratic form (Hayashi, 1985):

$$G(K_t, I_t) = \frac{b}{2} \left[\left(\frac{I_t}{K_t} \right) - c - e_t \right]^2 K_t \quad (5)$$

in which the error term e_t is an adjustment cost shock, observed by the firm but not by the econometrician, which may be serially correlated.

III. PROPOSED MODEL

A. Flexible Investment Procedure

The flexible investment design procedure we describe in this section is for investment that produce multiple product types whose demands may fluctuate from period to period according to a known distribution. To illustrate the procedure, we consider the case where the demand for each product is characterized by a finite discrete distribution, represented by a finite number of demand realization scenarios and probabilities of occurrence of each scenario. Demand for firm's product-outputs is assumed to be independent. The procedure can be easily extended to continuous and correlated demand distributions. Characterizing the demand distributions may be based on historical data and/or forecasts. We assume the following information is known:

The set of product types $P = \{p_1, p_2, \dots, p_P\}$ produced by the firm.

The demand distribution for each product p_i , as described by a set of demand realization scenarios $D_i = \{d_{i1}, d_{i2}, \dots, d_{in_i}\}$ and associated probabilities $\Pi_i = \{\pi_{i1}, \pi_{i2}, \dots, \pi_{in_i}\}$.

The basic steps of our procedure can be summarized as follows:

- 1) From the product demand and the price of capital goods distributions for each possible scenario, we obtain all condition alternatives for each demand scenario s .
- 2) From the individual product demand distributions, we obtain the probability of occurrence of each demand scenario s , (s).
- 3) For each demand scenario, we generate the corresponding optimal investment (when an optimal investment is computationally difficult to obtain, we use the heuristic procedure outlined later) and the corresponding stock of capital (K_t^*) and variable inputs (L_t^*) optimal.
- 4) Once all the investment have been generated, we evaluate each investment over the entire set of possible demand scenario and select the most flexible (for example, we choose the investment with the highest expected revenue of firm's revenue over all demand scenarios).

The procedure described below can be used when part demands are continuously distributed. Monte Carlo simulation is used to generate instances of product demands. The number of generated instances, or the sample size, must allow the simulation to reach a steady state. Details on constructing Monte Carlo simulations can be found in Law and Kelton (1994). Solution procedure:

Step 1: Read the distribution for the demand of each firm's product.

Step 2: Read sample size S (number of scenarios to be

evaluated).

Step 3: $I = 0$.

Step 4: $I = I + 1$. If $I > S$, then go to step 8.

Step 5: Generate demand revenues for each product by Monte Carlo simulation.

Step 6: Using demand revenues, generate the corresponding firm's revenue matrix.

Step 7: Generate an optimal investment for the current firm's revenue matrix (see section 3.2 for details). Go to back to step 4.

Step 8: Evaluate all generated investment and generate/choose the most flexible one (see section 3.2 for details).

Stop 9. Stop.

B. Flexible Investment Selection

The result of either the optimization model or the heuristic is an investment of the various firm's output demand. Once such an investment has been identified for every possible demand realization scenario, a decision must be made as to which of these investment is the most flexible. By definition, a flexible investment is one that maintains high firm's revenue despite changes in demand levels. Therefore, a possible measure of investment flexibility is the investment's expected firm's revenue over the range of feasible demand scenarios. This is a similar definition to that proposed by Rosenblatt and Kropp (1992). A procedure for identifying the most flexible investment, can then be obtained by (1) constructing the matrix $z(R(I), J)$, where $z(R(I), J)$ is the firm's revenue resulting from using an investment generated for demand scenario I when the actual demand corresponds to that of scenario J , and (2) selecting the most flexible investment by choosing, for example, the investment with the highest expected firm's revenue over all demand scenarios.

Note that in obtaining the firm's revenue of operating an investment that was designed for demand scenario I under scenario J , Bon and Cummins's procedure is solved in order to determine the optimal investment (see Bon and Cummins, 2001). The expected firm's revenue of each investment, $E[z(R(I))]$, is given by:

$$E[z(R(I))] = \sum [\pi(J) * z(R(I), J)] ; J \in S \quad (6)$$

where:

$\pi(J)$: the probability of occurrence of demand scenario J

S : the set of all feasible scenarios.

The above definition of a flexible investment, as the one with the highest average firm's revenue, is by no means unique. Alternative criteria for selecting the most flexible investment can be used. For example, a robustness criterion may, for certain facilities, be more appropriate. The investment whose revenue does not exceed the highest firm's revenue investment by a certain percentage the most number of times is chosen. This criterion is similar to the one used in Rosenblatt and Lee (1987). The robustness criterion can be particularly useful when probabilities are difficult to assign to different demand scenarios. A min-max criterion could also be used when the objective is to limit the revenue of the worst case scenario.

Both criteria, expected cost and robustness, do not, however, account for variance. An investment with high firm's revenue may have high variance if its performance

fluctuates significantly from one demand scenario to another. This can be unacceptable if the firm's revenue is excessively high for one or more of these scenarios. Variance can be accounted for by using a combined mean-variance criterion. Alternatively, certain investment may be disqualified from further consideration if their associated cost exceeds a certain level for one or more demand scenarios.

Finally, we should note that in identifying the most flexible investment, we have restricted in our current procedure our attention to the set of the best investment selected for different scenarios. In doing so, we may have overlooked an investment with a higher overall expected firm's revenue that is not necessarily optimal for any scenario. This limitation can be overcome by directly solving for the investment with the highest expected firm's revenue. A procedure that implements this approach is described in Appendix 2. Extensive experimentation with both procedures indicates that the second procedure yields investment with slightly higher expected firm's revenue. The proposed procedure is, however, more general since it allows for alternative criteria (other than firm's revenue) for measuring investment flexibility to be used.

IV. NUMERIC EXAMPLE

A simple example that illustrates the various steps in generating a flexible investment is described in this section for the discrete distribution of product demand. There are two product types (see Table 1), with demand for each product type being either high (H), medium (M), or low (L) (see Table 2). This results in 9 possible demand scenarios. For each demand scenario, the best investment and its corresponding firm's revenue are calculated. The results are listed in Table 3. The performance of each of these selected investment is then evaluated under all possible scenarios and the expected performance of each investment is calculated. The results are given in Table 4. The investment with the highest expected firm's revenue is then selected. In this case, the highest expected firm's revenue is the one initially selected for demand scenario 5 (MM).

TABLE 1. FIRM'S PRODUCT DEMAND UNDER DIFFERENT STATES

| Parts | Low | Medium | High |
|-------|-----|--------|------|
| P1 | 50 | 80 | 170 |
| P2 | 70 | 140 | 170 |

TABLE 2. PROBABILITY OF OCCURRENCE FOR DIFFERENT DEMAND STATES

| Sates | Probability of occurrence |
|------------|---------------------------|
| Low (L) | 0.2 |
| Medium (M) | 0.5 |
| High (H) | 0.3 |

TABLE 3. GENERATED INVESTMENT AND FIRM'S REVENUE PER SCENARIOS

| Scenario | Scenario probability | Selected Investment | Firm's Revenue |
|----------|----------------------|---------------------|----------------|
| LL(1) | 0.04 | \$610 | \$1650 |
| LM(2) | 0.1 | \$930 | \$1900 |
| LH(3) | 0.06 | \$1050 | \$1750 |
| ML(4) | 0.1 | \$720 | \$2000 |
| MM(5) | 0.25 | \$1025 | \$2100 |
| MH(6) | 0.15 | \$1200 | \$1800 |
| HL(7) | 0.06 | \$1150 | \$1750 |
| HM(8) | 0.15 | \$1400 | \$1950 |
| HH(9) | 0.09 | \$1550 | \$2050 |

TABLE 4. EXPECTED FIRM'S REVENUE FOR SELECTED INVESTMENT

| Scenario | Scenario probability | z(R(I), J) | | | | | | | | | E(z(R(I))) |
|----------|----------------------|------------|------|------|------|------|------|------|------|------|------------|
| | | 0,04 | 0,1 | 0,06 | 0,1 | 0,25 | 0,15 | 0,06 | 0,15 | 0,09 | |
| I=1 | 0,04 | 1650 | 1760 | 1790 | 1450 | 1590 | 1680 | 1770 | 1890 | 1970 | 1710,9 |
| I=2 | 0,1 | 1750 | 1900 | 1960 | 1780 | 1800 | 1870 | 1910 | 1930 | 1970 | 1867,5 |
| I=3 | 0,06 | 1500 | 1650 | 1750 | 1650 | 1800 | 1870 | 1880 | 1900 | 2010 | 1804,2 |
| I=4 | 0,1 | 1700 | 1780 | 1850 | 2000 | 2010 | 2030 | 2050 | 2060 | 2070 | 1982,3 |
| I=5 | 0,25 | 1740 | 1850 | 1900 | 2000 | 2100 | 2110 | 2140 | 2160 | 2180 | 2058,7 |
| I=6 | 0,15 | 1700 | 1850 | 1900 | 1730 | 1750 | 1800 | 1870 | 1890 | 1900 | 1814,2 |
| I=7 | 0,06 | 1840 | 1860 | 1870 | 1690 | 1730 | 1740 | 1750 | 1900 | 1960 | 1800,7 |
| I=8 | 0,15 | 1790 | 1820 | 1900 | 1730 | 1760 | 1800 | 1850 | 1950 | 2000 | 1834,1 |
| I=9 | 0,09 | 1840 | 1850 | 1900 | 1800 | 1810 | 1850 | 1900 | 1950 | 2050 | 1873,6 |

It is interesting to note that the performance differential between each of the selected investment for the different scenarios is not very significant. This is made evident by the similarity between the different rows of Table 3. We believe that this is due to the highly distributed nature of the generated investment and to the existing high level of demand. It is easy to see that the flexible investment outperform the traditional investment under all 9 demand scenarios. More importantly, the flexible investment is more robust in the face of variability. A traditional investment that performs well under one scenario can, indeed, perform poorly under another. This is illustrated in Table 5, where the revenues of $E(z(R(I)))$ are shown for both the traditional and flexible investment. Note that while for the flexible investment the revenue of $E(z(R(I)))$ vary very little, wider fluctuations are experienced by the traditional investment. In particular, we found that the desirability for investment duplication and distribution tends to increase with increases in either product demand variability.

TABLE 5. EXPECTED FIRM'S REVENUE COMPARISON BETWEEN FLEXIBLE INVESTMENT AND TRADITIONAL INVESTMENT

| Scenario | E(z(R(I)) for Flexible | E(z(R(I)) for Traditional |
|----------|------------------------|---------------------------|
| I=1 | 1710,9 | 1700,0 |
| I=2 | 1867,5 | 1820,2 |
| I=3 | 1804,2 | 1789,3 |
| I=4 | 1982,3 | 1930,5 |
| I=5 | 2058,7 | 2000,0 |
| I=6 | 1814,2 | 1802,4 |
| I=7 | 1800,7 | 1790,3 |
| I=8 | 1834,1 | 1810,5 |
| I=9 | 1873,6 | 1854,2 |

Finally, since there seems to be little difference in performance between the flexible investment generated for different demand scenarios, it could be argued that there is no

need to optimize the investment for each demand scenario. Instead, provided a reasonable amount of investment duplication. This would, in fact, greatly simplify our design procedure since the investment would have to generated only once. Only one investment strategy would then be optimized for different demand scenarios. Further experimentation is however needed before the general effectiveness of such an approach is established.

V. CONCLUSION

In this paper, we addressed the problem of designing flexible investment where product demands are subject to variability. New procedures for designing these investment were proposed. Heuristic solution methods were presented for various design and operation assumptions. The new procedures explicitly account for the stochastic nature of product demands is very simple. Numerical examples seem to indicate that selected investment generally results in a more flexible and robust layout than the one resulting from traditional. On the other hand, in the absence of investment duplication, the need for carefully selecting the most flexible investment becomes more important and the procedures presented here become more valuable. Additional experimentation is, however, needed in order to generalize these observations.

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